# 9.3 OPERATIONS WITH RADICALS

**In this section**

- Adding and Subtracting Radicals
- Multiplying Radicals
- Conjugates

In this section we will use the ideas of Section 9.1 in performing arithmetic operations with radical expressions.

## Adding and Subtracting Radicals

To find the sum of $\sqrt{2}$ and $\sqrt{3}$, we can use a calculator to get $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.732$. (The symbol $\approx$ means "is approximately equal to.") We can then add the decimal numbers and get

$$\sqrt{2} + \sqrt{3} \approx 1.414 + 1.732 = 3.146.$$

We cannot write an exact decimal form for $\sqrt{2} + \sqrt{3}$; the number 3.146 is an approximation of $\sqrt{2} + \sqrt{3}$. To represent the exact value of $\sqrt{2} + \sqrt{3}$, we just use the form $\sqrt{2} + \sqrt{3}$. This form cannot be simplified any further. However, a sum of like radicals can be simplified. **Like radicals** are radicals that have the same index and the same radicand.

To simplify the sum $3\sqrt{2} + 5\sqrt{2}$, we can use the fact that $3x + 5x = 8x$ is true for any value of $x$. Substituting $\sqrt{2}$ for $x$ gives us $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$. So like radicals can be combined just as like terms are combined.

**EXAMPLE 1** **Adding and subtracting like radicals**

Simplify the following expressions. Assume the variables represent positive numbers.

**a)** $3\sqrt{5} + 4\sqrt{5}$  **b)** $\sqrt[4]{w} - 6\sqrt[4]{w}$

**c)** $\sqrt{3} + \sqrt{5} - 4\sqrt{3} + 6\sqrt{5}$  **d)** $3\sqrt[3]{6x} + 2\sqrt[3]{x} + \sqrt[3]{6x} + \sqrt[3]{x}$

**Solution**

**a)** $3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$  **b)** $\sqrt[4]{w} - 6\sqrt[4]{w} = -5\sqrt[4]{w}$

**c)** $\sqrt{3} + \sqrt{5} - 4\sqrt{3} + 6\sqrt{5} = -3\sqrt{3} + 7\sqrt{5}$

**d)** $3\sqrt[3]{6x} + 2\sqrt[3]{x} + \sqrt[3]{6x} + \sqrt[3]{x} = 4\sqrt[3]{6x} + 3\sqrt[3]{x}$  Only like radicals are combined. ■

Remember that *only radicals with the same index and same radicand can be combined by addition or subtraction.* If the radicals are not in simplified form, then they must be simplified before you can determine whether they can be combined.

**EXAMPLE 2** **Simplifying radicals before combining**

Perform the indicated operations. Assume the variables represent positive numbers.

**a)** $\sqrt{8} + \sqrt{18}$  **b)** $\sqrt{\frac{1}{5}} + \sqrt{20}$

**c)** $\sqrt{2x^3} - \sqrt{4x^2} + 5\sqrt{18x^3}$  **d)** $\sqrt[3]{16x^4y^3} - \sqrt[3]{54x^4y^3}$

**Solution**

**a)**
$$\begin{aligned}\sqrt{8} + \sqrt{18} &= \sqrt{4}\cdot\sqrt{2} + \sqrt{9}\cdot\sqrt{2}\\ &= 2\sqrt{2} + 3\sqrt{2} \quad \text{Simplify each radical.}\\ &= 5\sqrt{2} \quad \text{Add like radicals.}\end{aligned}$$

Note that $\sqrt{8} + \sqrt{18} \neq \sqrt{26}$.

**calculator close-up**

Check that

$\sqrt{8} + \sqrt{18} = 5\sqrt{2}$.

```
√(8)+√(18)
         7.071067812
5√(2)
         7.071067812
```

**b)** $\sqrt{\frac{1}{5}} + \sqrt{20} = \frac{\sqrt{5}}{5} + 2\sqrt{5}$ Because $\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ and $\sqrt{20} = 2\sqrt{5}$

$= \frac{\sqrt{5}}{5} + \frac{10\sqrt{5}}{5}$ Use the LCD of 5.

$= \frac{11\sqrt{5}}{5}$

**c)** $\sqrt{2x^3} - \sqrt{4x^2} + 5\sqrt{18x^3} = \sqrt{x^2} \cdot \sqrt{2x} - 2x + 5 \cdot \sqrt{9x^2} \cdot \sqrt{2x}$

$= x\sqrt{2x} - 2x + 15x\sqrt{2x}$ Simplify each radical.

$= 16x\sqrt{2x} - 2x$ Add like radicals only.

**d)** $\sqrt[3]{16x^4y^3} - \sqrt[3]{54x^4y^3} = \sqrt[3]{8x^3y^3} \cdot \sqrt[3]{2x} - \sqrt[3]{27x^3y^3} \cdot \sqrt[3]{2x}$

$= 2xy\sqrt[3]{2x} - 3xy\sqrt[3]{2x}$ Simplify each radical.

$= -xy\sqrt[3]{2x}$ ■

## Multiplying Radicals

We have already multiplied radicals in Section 9.2, when we rationalized denominators. The product rule for radicals, $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$, allows multiplication of radicals with the same index, such as

$$\sqrt{5} \cdot \sqrt{3} = \sqrt{15}, \quad \sqrt[3]{2} \cdot \sqrt[3]{5} = \sqrt[3]{10}, \quad \text{and} \quad \sqrt[5]{x^2} \cdot \sqrt[5]{x} = \sqrt[5]{x^3}.$$

**CAUTION** The product rule does not allow multiplication of radicals that have different indices. We cannot use the product rule to multiply $\sqrt{2}$ and $\sqrt[3]{5}$.

**helpful hint**

Students often write

$\sqrt{15} \cdot \sqrt{15} = \sqrt{225} = 15.$

Although this is correct, you should get used to the idea that

$\sqrt{15} \cdot \sqrt{15} = 15.$

Because of the definition of a square root, $\sqrt{a} \cdot \sqrt{a} = a$ for any positive number $a$.

**EXAMPLE 3**

**Multiplying radicals with the same index**

Multiply and simplify the following expressions. Assume the variables represent positive numbers.

**a)** $5\sqrt{6} \cdot 4\sqrt{3}$

**b)** $\sqrt{3a^2} \cdot \sqrt{6a}$

**c)** $\sqrt[3]{4} \cdot \sqrt[3]{4}$

**d)** $\sqrt[4]{\frac{x^3}{2}} \cdot \sqrt[4]{\frac{x^2}{4}}$

### Solution

**a)** $5\sqrt{6} \cdot 4\sqrt{3} = 5 \cdot 4 \cdot \sqrt{6} \cdot \sqrt{3}$

$= 20\sqrt{18}$ Product rule for radicals

$= 20 \cdot 3\sqrt{2}$ $\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

$= 60\sqrt{2}$

**b)** $\sqrt{3a^2} \cdot \sqrt{6a} = \sqrt{18a^3}$ Product rule for radicals

$= \sqrt{9a^2} \cdot \sqrt{2a}$

$= 3a\sqrt{2a}$ Simplify.

**c)** $\sqrt[3]{4} \cdot \sqrt[3]{4} = \sqrt[3]{16}$

$= \sqrt[3]{8} \cdot \sqrt[3]{2}$ Simplify.

$= 2\sqrt[3]{2}$

**d)** $\sqrt[4]{\frac{x^3}{2}} \cdot \sqrt[4]{\frac{x^2}{4}} = \sqrt[4]{\frac{x^5}{8}}$ Product rule for radicals

$= \frac{\sqrt[4]{x^4} \cdot \sqrt[4]{x}}{\sqrt[4]{8}}$ Simplify

$= \frac{x\sqrt[4]{x}}{\sqrt[4]{8}}$

$= \frac{x\sqrt[4]{x} \cdot \sqrt[4]{2}}{\sqrt[4]{8} \cdot \sqrt[4]{2}}$ Rationalize the denominator.

$= \frac{x\sqrt[4]{2x}}{2}$ $\sqrt[4]{8} \cdot \sqrt[4]{2} = \sqrt[4]{16} = 2$ ■

We find a product such as $3\sqrt{2}(4\sqrt{2} - \sqrt{3})$ by using the distributive property as we do when multiplying a monomial and a binomial. A product such as $(2\sqrt{3} + \sqrt{5})(3\sqrt{3} - 2\sqrt{5})$ can be found by using FOIL as we do for the product of two binomials.

**EXAMPLE 4** **Multiplying radicals**

Multiply and simplify.

**a)** $3\sqrt{2}(4\sqrt{2} - \sqrt{3})$ **b)** $\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a^2})$

**c)** $(2\sqrt{3} + \sqrt{5})(3\sqrt{3} - 2\sqrt{5})$ **d)** $(3 + \sqrt{x - 9})^2$

## Solution

**a)** $3\sqrt{2}(4\sqrt{2} - \sqrt{3}) = 3\sqrt{2} \cdot 4\sqrt{2} - 3\sqrt{2} \cdot \sqrt{3}$ Distributive property

$= 12 \cdot 2 - 3\sqrt{6}$ Because $\sqrt{2} \cdot \sqrt{2} = 2$ and $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

$= 24 - 3\sqrt{6}$

**b)** $\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a^2}) = \sqrt[3]{a^2} - \sqrt[3]{a^3}$ Distributive property

$= \sqrt[3]{a^2} - a$

**c)** $(2\sqrt{3} + \sqrt{5})(3\sqrt{3} - 2\sqrt{5})$

$= \overbrace{2\sqrt{3} \cdot 3\sqrt{3}}^{F} - \overbrace{2\sqrt{3} \cdot 2\sqrt{5}}^{O} + \overbrace{\sqrt{5} \cdot 3\sqrt{3}}^{I} - \overbrace{\sqrt{5} \cdot 2\sqrt{5}}^{L}$

$= 18 - 4\sqrt{15} + 3\sqrt{15} - 10$

$= 8 - \sqrt{15}$ Combine like radicals.

**d)** To square a sum, we use $(a + b)^2 = a^2 + 2ab + b^2$:

$$(3 + \sqrt{x - 9})^2 = 3^2 + 2 \cdot 3\sqrt{x - 9} + (\sqrt{x - 9})^2$$
$$= 9 + 6\sqrt{x - 9} + x - 9$$
$$= x + 6\sqrt{x - 9}$$ ■

In the next example we multiply radicals that have different indices.

**EXAMPLE 5** **Multiplying radicals with different indices**

Write each product as a single radical expression.

**a)** $\sqrt[3]{2} \cdot \sqrt[4]{2}$ **b)** $\sqrt[3]{2} \cdot \sqrt{3}$

**Solution**

**a)**
$$\begin{aligned} \sqrt[3]{2} \cdot \sqrt[4]{2} &= 2^{1/3} \cdot 2^{1/4} && \text{Write in exponential notation.} \\ &= 2^{7/12} && \text{Product rule for exponents: } \tfrac{1}{3} + \tfrac{1}{4} = \tfrac{7}{12} \\ &= \sqrt[12]{2^7} && \text{Write in radical notation.} \\ &= \sqrt[12]{128} \end{aligned}$$

**b)**
$$\begin{aligned} \sqrt[3]{2} \cdot \sqrt{3} &= 2^{1/3} \cdot 3^{1/2} && \text{Write in exponential notation.} \\ &= 2^{2/6} \cdot 3^{3/6} && \text{Write the exponents with the LCD of 6.} \\ &= \sqrt[6]{2^2} \cdot \sqrt[6]{3^3} && \text{Write in radical notation.} \\ &= \sqrt[6]{2^2 \cdot 3^3} && \text{Product rule for radicals} \\ &= \sqrt[6]{108} && 2^2 \cdot 3^3 = 4 \cdot 27 = 108 \end{aligned}$$

**calculator close-up**

Check that

$\sqrt[3]{2} \cdot \sqrt[4]{2} = \sqrt[12]{128}.$

```
2^(1/3)*2^(1/4)
        1.498307077
128^(1/12)
        1.498307077
```

**CAUTION** Because the bases in $2^{1/3} \cdot 2^{1/4}$ are identical, we can add the exponents [Example 5(a)]. Because the bases in $2^{2/6} \cdot 3^{3/6}$ are not the same, we cannot add the exponents [Example 5(b)]. Instead, we write each factor as a sixth root and use the product rule for radicals.

## Conjugates

**helpful hint**

The word "conjugate" is used in many contexts in mathematics. According to the dictionary, conjugate means joined together, especially as in a pair.

Recall the special product rule $(a + b)(a - b) = a^2 - b^2$. The product of the sum $4 + \sqrt{3}$ and the difference $4 - \sqrt{3}$ can be found by using this rule:

$$(4 + \sqrt{3})(4 - \sqrt{3}) = 4^2 - (\sqrt{3})^2 = 16 - 3 = 13$$

The product of the irrational number $4 + \sqrt{3}$ and the irrational number $4 - \sqrt{3}$ is the rational number 13. For this reason the expressions $4 + \sqrt{3}$ and $4 - \sqrt{3}$ are called **conjugates** of one another. We will use conjugates in Section 9.4 to rationalize some denominators.

**EXAMPLE 6** **Multiplying conjugates**

Find the products. Assume the variables represent positive real numbers.

**a)** $(2 + 3\sqrt{5})(2 - 3\sqrt{5})$

**b)** $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$

**c)** $(\sqrt{2x} - \sqrt{y})(\sqrt{2x} + \sqrt{y})$

**Solution**

**a)**
$$\begin{aligned} (2 + 3\sqrt{5})(2 - 3\sqrt{5}) &= 2^2 - (3\sqrt{5})^2 && (a + b)(a - b) = a^2 - b^2 \\ &= 4 - 45 && (3\sqrt{5})^2 = 9 \cdot 5 = 45 \\ &= -41 \end{aligned}$$

**b)**
$$\begin{aligned} (\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) &= 3 - 2 \\ &= 1 \end{aligned}$$

**c)** $(\sqrt{2x} - \sqrt{y})(\sqrt{2x} + \sqrt{y}) = 2x - y$

## WARM-UPS

**True or false? Explain your answer.**

1. $\sqrt{3} + \sqrt{3} = \sqrt{6}$
2. $\sqrt{8} + \sqrt{2} = 3\sqrt{2}$
3. $2\sqrt{3} \cdot 3\sqrt{3} = 6\sqrt{3}$
4. $\sqrt[3]{2} \cdot \sqrt[3]{2} = 2$
5. $2\sqrt{5} \cdot 3\sqrt{2} = 6\sqrt{10}$
6. $2\sqrt{5} + 3\sqrt{5} = 5\sqrt{10}$
7. $\sqrt{2}(\sqrt{3} - \sqrt{2}) = \sqrt{6} - 2$
8. $\sqrt{12} = 2\sqrt{6}$
9. $(\sqrt{2} + \sqrt{3})^2 = 2 + 3$
10. $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) = 1$

## 9.3 EXERCISES

***Reading and Writing*** *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What are like radicals?
2. How do we combine like radicals?
3. Does the product rule allow multiplication of unlike radicals?
4. How do we multiply radicals of different indices?

*All variables in the following exercises represent positive numbers.*

*Simplify the sums and differences. Give exact answers. See Example 1.*

5. $\sqrt{3} - 2\sqrt{3}$
6. $\sqrt{5} - 3\sqrt{5}$
7. $5\sqrt{7x} + 4\sqrt{7x}$
8. $3\sqrt{6a} + 7\sqrt{6a}$
9. $2\sqrt[3]{2} + 3\sqrt[3]{2}$
10. $\sqrt[3]{4} + 4\sqrt[3]{4}$
11. $\sqrt{3} - \sqrt{5} + 3\sqrt{3} - \sqrt{5}$
12. $\sqrt{2} - 5\sqrt{3} - 7\sqrt{2} + 9\sqrt{3}$
13. $\sqrt[3]{2} + \sqrt[3]{x} - \sqrt[3]{2} + 4\sqrt[3]{x}$
14. $\sqrt[3]{5y} - 4\sqrt[3]{5y} + \sqrt[3]{x} + \sqrt[3]{x}$
15. $\sqrt[3]{x} - \sqrt{2x} + \sqrt[3]{x}$
16. $\sqrt[3]{ab} + \sqrt{a} + 5\sqrt{a} + \sqrt[3]{ab}$

*Simplify each expression. Give exact answers. See Example 2.*

17. $\sqrt{8} + \sqrt{28}$
18. $\sqrt{12} + \sqrt{24}$
19. $\sqrt{8} + \sqrt{18}$
20. $\sqrt{12} + \sqrt{27}$
21. $\sqrt{45} - \sqrt{20}$
22. $\sqrt{50} - \sqrt{32}$
23. $\sqrt{2} - \sqrt{8}$
24. $\sqrt{20} - \sqrt{125}$
25. $\dfrac{\sqrt{2}}{2} + \sqrt{2}$
26. $\dfrac{\sqrt{3}}{3} - \sqrt{3}$
27. $\sqrt{80} + \sqrt{\dfrac{1}{5}}$
28. $\sqrt{32} + \sqrt{\dfrac{1}{2}}$
29. $\sqrt{45x^3} - \sqrt{18x^2} + \sqrt{50x^2} - \sqrt{20x^3}$
30. $\sqrt{12x^5} - \sqrt{18x} - \sqrt{300x^5} + \sqrt{98x}$
31. $\sqrt[3]{24} + \sqrt[3]{81}$
32. $\sqrt[3]{24} + \sqrt[3]{375}$
33. $\sqrt[4]{48} - \sqrt[4]{243}$
34. $\sqrt[5]{64} + \sqrt[5]{2}$

**35.** $\sqrt[3]{54t^4y^3} - \sqrt[3]{16t^4y^3}$

**36.** $\sqrt[3]{2000w^2z^5} - \sqrt[3]{16w^2z^5}$

*Simplify the products. Give exact answers. See Examples 3 and 4.*

**37.** $\sqrt{3} \cdot \sqrt{5}$

**38.** $\sqrt{5} \cdot \sqrt{7}$

**39.** $2\sqrt{5} \cdot 3\sqrt{10}$

**40.** $(3\sqrt{2})(-4\sqrt{10})$

**41.** $2\sqrt{7a} \cdot 3\sqrt{2a}$

**42.** $2\sqrt{5c} \cdot 5\sqrt{5}$

**43.** $\sqrt[4]{9} \cdot \sqrt[4]{27}$

**44.** $\sqrt[3]{5} \cdot \sqrt[3]{100}$

**45.** $(2\sqrt{3})^2$

**46.** $(-4\sqrt{2})^2$

**47.** $\sqrt[3]{\dfrac{4x^2}{3}} \cdot \sqrt[3]{\dfrac{2x^2}{3}}$

**48.** $\sqrt[4]{\dfrac{4x^2}{5}} \cdot \sqrt[4]{\dfrac{4x^3}{25}}$

**49.** $2\sqrt{3}(\sqrt{6} + 3\sqrt{3})$

**50.** $2\sqrt{5}(\sqrt{3} + 3\sqrt{5})$

**51.** $\sqrt{5}(\sqrt{10} - 2)$

**52.** $\sqrt{6}(\sqrt{15} - 1)$

**53.** $\sqrt[3]{3t}(\sqrt[3]{9t} - \sqrt[3]{t^2})$

**54.** $\sqrt[3]{2}(\sqrt[3]{12x} - \sqrt[3]{2x})$

**55.** $(\sqrt{3} + 2)(\sqrt{3} - 5)$

**56.** $(\sqrt{5} + 2)(\sqrt{5} - 6)$

**57.** $(\sqrt{11} - 3)(\sqrt{11} + 3)$

**58.** $(\sqrt{2} + 5)(\sqrt{2} + 5)$

**59.** $(2\sqrt{5} - 7)(2\sqrt{5} + 4)$

**60.** $(2\sqrt{6} - 3)(2\sqrt{6} + 4)$

**61.** $(2\sqrt{3} - \sqrt{6})(\sqrt{3} + 2\sqrt{6})$

**62.** $(3\sqrt{3} - \sqrt{2})(\sqrt{2} + \sqrt{3})$

*Write each product as a single radical expression. See Example 5.*

**63.** $\sqrt[3]{3} \cdot \sqrt{3}$

**64.** $\sqrt{3} \cdot \sqrt[4]{3}$

**65.** $\sqrt[3]{5} \cdot \sqrt[4]{5}$

**66.** $\sqrt[3]{2} \cdot \sqrt[5]{2}$

**67.** $\sqrt[3]{2} \cdot \sqrt{5}$

**68.** $\sqrt{6} \cdot \sqrt[3]{2}$

**69.** $\sqrt[3]{2} \cdot \sqrt[4]{3}$

**70.** $\sqrt[3]{3} \cdot \sqrt[4]{2}$

*Find the product of each pair of conjugates. See Example 6.*

**71.** $(\sqrt{3} - 2)(\sqrt{3} + 2)$

**72.** $(7 - \sqrt{3})(7 + \sqrt{3})$

**73.** $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$

**74.** $(\sqrt{6} + \sqrt{5})(\sqrt{6} - \sqrt{5})$

**75.** $(2\sqrt{5} + 1)(2\sqrt{5} - 1)$

**76.** $(3\sqrt{2} - 4)(3\sqrt{2} + 4)$

**77.** $(3\sqrt{2} + \sqrt{5})(3\sqrt{2} - \sqrt{5})$

**78.** $(2\sqrt{3} - \sqrt{7})(2\sqrt{3} + \sqrt{7})$

**79.** $(5 - 3\sqrt{x})(5 + 3\sqrt{x})$

**80.** $(4\sqrt{y} + 3\sqrt{z})(4\sqrt{y} - 3\sqrt{z})$

*Simplify each expression.*

**81.** $\sqrt{300} + \sqrt{3}$

**82.** $\sqrt{50} + \sqrt{2}$

**83.** $2\sqrt{5} \cdot 5\sqrt{6}$

**84.** $3\sqrt{6} \cdot 5\sqrt{10}$

**85.** $(3 + 2\sqrt{7})(\sqrt{7} - 2)$

**86.** $(2 + \sqrt{7})(\sqrt{7} - 2)$

**87.** $4\sqrt{w} \cdot 4\sqrt{w}$

**88.** $3\sqrt{m} \cdot 5\sqrt{m}$

**89.** $\sqrt{3x^3} \cdot \sqrt{6x^2}$

**90.** $\sqrt{2t^5} \cdot \sqrt{10t^4}$

**91.** $\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{8}} + \dfrac{1}{\sqrt{18}}$

**92.** $\dfrac{1}{\sqrt{3}} + \sqrt{\dfrac{1}{3}} - \sqrt{3}$

**93.** $(2\sqrt{5} + \sqrt{2})(3\sqrt{5} - \sqrt{2})$

**94.** $(3\sqrt{2} - \sqrt{3})(2\sqrt{2} + 3\sqrt{3})$

**95.** $\dfrac{\sqrt{2}}{3} + \dfrac{\sqrt{2}}{5}$

**96.** $\dfrac{\sqrt{2}}{4} + \dfrac{\sqrt{3}}{5}$

**97.** $(5 + 2\sqrt{2})(5 - 2\sqrt{2})$

**98.** $(3 - 2\sqrt{7})(3 + 2\sqrt{7})$

**99.** $(3 + \sqrt{x})^2$

**100.** $(1 - \sqrt{x})^2$

**101.** $(5\sqrt{x} - 3)^2$

**102.** $(3\sqrt{a} + 2)^2$

**103.** $(1 + \sqrt{x + 2})^2$

**104.** $(\sqrt{x - 1} + 1)^2$

**105.** $\sqrt{4w} - \sqrt{9w}$

**106.** $10\sqrt{m} - \sqrt{16m}$

**107.** $2\sqrt{a^3} + 3\sqrt{a^3} - 2a\sqrt{4a}$

**108.** $5\sqrt{w^2y} - 7\sqrt{w^2y} + 6\sqrt{w^2y}$

**109.** $\sqrt{x^5} + 2x\sqrt{x^3}$

**110.** $\sqrt{8x^3} + \sqrt{50x^3} - x\sqrt{2x}$

**111.** $\sqrt[3]{-16x^4} + 5x\sqrt[3]{54x}$

**112.** $\sqrt[3]{3x^5y^7} - \sqrt[3]{24x^5y^7}$

**113.** $\sqrt[3]{\dfrac{y^7}{4x}}$

**114.** $\sqrt[4]{\dfrac{16}{9z^3}}$

**115.** $\sqrt[3]{\dfrac{x}{5}} \cdot \sqrt[3]{\dfrac{x^5}{5}}$

**116.** $\sqrt[4]{a^3}(\sqrt[4]{a} - \sqrt[4]{a^5})$

**117.** $\sqrt[3]{2x} \cdot \sqrt{2x}$

**118.** $\sqrt[3]{2m} \cdot \sqrt[4]{2n}$

*In Exercises 119–122, solve each problem.*

**119.** ***Area of a rectangle.*** Find the exact area of a rectangle that has a length of $\sqrt{6}$ feet and a width of $\sqrt{3}$ feet.

**120.** ***Volume of a cube.*** Find the exact volume of a cube with sides of length $\sqrt{3}$ meters.

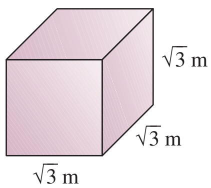


**FIGURE FOR EXERCISE 120**

**121.** ***Area of a trapezoid.*** Find the exact area of a trapezoid with a height of $\sqrt{6}$ feet and bases of $\sqrt{3}$ feet and $\sqrt{12}$ feet.

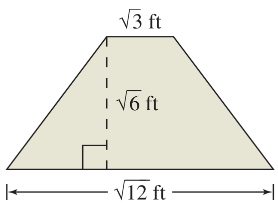


**FIGURE FOR EXERCISE 121**

**122.** ***Area of a triangle.*** Find the exact area of a triangle with a base of $\sqrt{30}$ meters and a height of $\sqrt{6}$ meters.

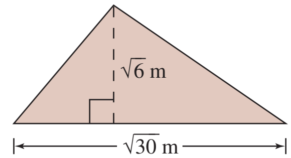


**FIGURE FOR EXERCISE 122**

## GETTING MORE INVOLVED

**123.** ***Discussion.*** Is $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$ for all values of $a$ and $b$?

**124.** ***Discussion.*** Which of the following equations are identities? Explain your answers.

**a)** $\sqrt{9x} = 3\sqrt{x}$

**b)** $\sqrt{9 + x} = 3 + \sqrt{x}$

**c)** $\sqrt{x - 4} = \sqrt{x} - 2$

**d)** $\sqrt{\dfrac{x}{4}} = \dfrac{\sqrt{x}}{2}$

**125.** ***Exploration.*** Because 3 is the square of $\sqrt{3}$, a binomial such as $y^2 - 3$ is a difference of two squares.

**a)** Factor $y^2 - 3$ and $2a^2 - 7$ using radicals.

**b)** Use factoring with radicals to solve the equations $x^2 - 8 = 0$ and $3y^2 - 11 = 0$.

**c)** Assuming $a$ and $b$ are positive real numbers, solve the equations $x^2 - a = 0$ and $ax^2 - b = 0$.

# 9.4 MORE OPERATIONS WITH RADICALS

**In this section**

- Dividing Radicals
- Rationalizing the Denominator
- Powers of Radical Expressions

In this section you will continue studying operations with radicals. We learn to rationalize some denominators that are different from those rationalized in Section 9.2.

## Dividing Radicals

In Section 9.3 you learned how to add, subtract, and multiply radical expressions. To divide two radical expressions, simply write the quotient as a ratio and then simplify, as we did in Section 9.2. In general, we have

$$\sqrt[n]{a} \div \sqrt[n]{b} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}},$$

provided that all expressions represent real numbers. Note that the quotient rule is applied only to radicals that have the same index.