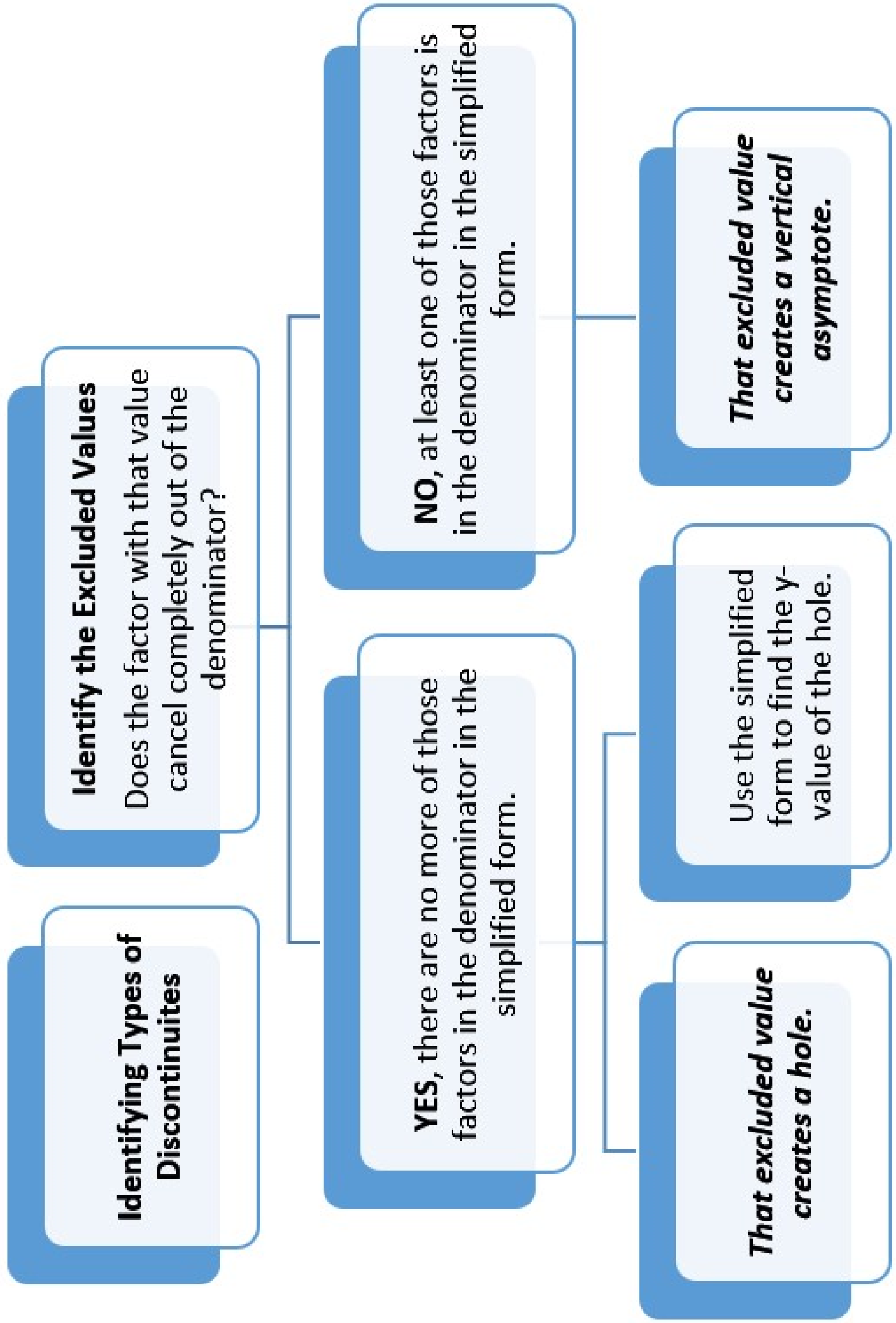

<u>**Discontinuities**</u> occur when the denominator = ____

Identify the excluded values and find the domain.

$f(x)=\dfrac{x^2-4x-21}{x^2-9x-36}$ $\qquad f(x)=\dfrac{3x^2+x-2}{x^2+2x-3}$ $\qquad f(x)=\dfrac{x+8}{x^2-x-6}$

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2 Types of Discontinuities:

- __________________ - creates a hole in the graph. Algebraically occurs when the factor containing the discontinuity is a ___________________________.
- __________________ - creates a vertical asymptote in the graph. Algebraically occurs when the discontinuous factor is not a ____________________________.

<u>Vertical Asymptotes</u>—To find, set the denominator equal to zero and solve.

Example: $f(x)=\dfrac{3x}{x^2-3x+2}$

<u>Hole</u>— If the factor x - b is in both the numerator and denominator, a hole occurs as x = b.

Example: $y=\dfrac{-6x^2+4x}{3x-2}$

**When *x* - *b* is a factor of both the numerator and the denominator AND has a greater multiplicity in the denominator, then *x* = *b* is a vertical asymptote and not a hole.

Example: $y=\dfrac{x^2(x-5)}{5x^3}$

Horizontal Asymptotes — Rational Functions

Let $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions in standard form with no common factors other than 1. The graph of f has at most one horizontal asymptote.

- If degree of p > degree of q, there is no horizontal asymptote.
- If degree of p < degree of q, the horizontal asymptote is the line $y = 0$.
- If degree of p = degree of q, the horizontal asymptote is the line

$$y = \frac{\text{leading coefficient of } p}{\text{leading coefficient of } q}.$$

Situation	Example 1	Example 2
Degree of p < Degree of q Horizontal Asymptote is y = 0		
Degree of p = Degree of q Horizontal Asymptote is y = L.C./L.C.		
Degree of p > Degree of q No Horizontal Asymptote Check for Slant Asymptote		

Identify all asymptotes for the graphs of the following rationals.

$$f(x) = \frac{x^3}{x^2 - x - 20}$$

$$f(x) = \frac{2x - 1}{x + 1}$$

Slant Asymptotes - Occurs when a rational function has a denominator with a degree of ______________________, and the degree of the numerator is ______________________ more than the degree of the denominator.

To find the equation of the slant asymptote, use division.

Find the slant asymptotes for the following.

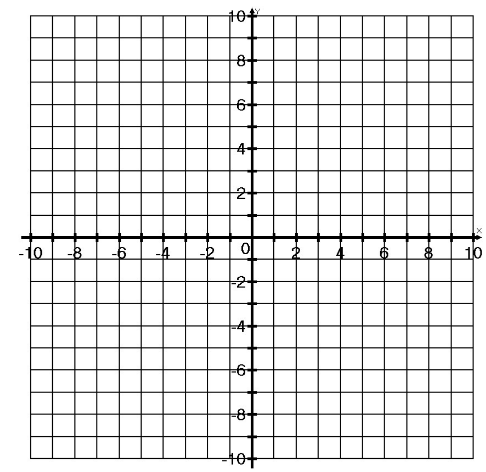
$$f(x) = \frac{x^2 - x}{x + 1}$$

$$f(x) = \frac{x^2 - x - 2}{x - 1}$$

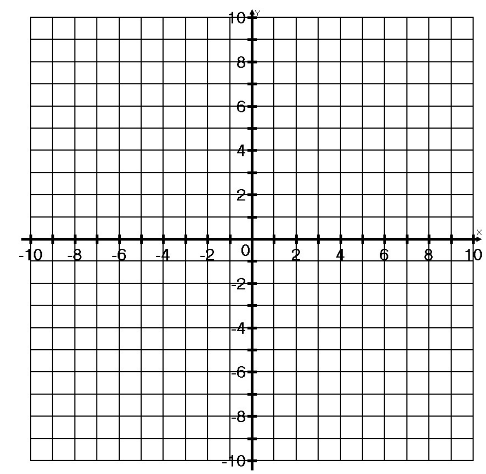
$$f(x) = \frac{3x^2 + 1}{x}$$

Identify all asymptotes and holes. Then, sketch a graph.

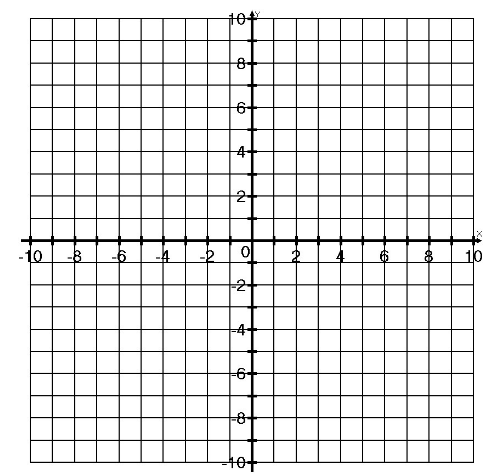
$$f(x) = \frac{3x - 28 + x^2}{x^2 + 12x + 35}$$



$$y = \frac{x^2 - 5x - 14}{2x^3 + 14x^2 + 20x}$$



$$y = \frac{3x^2 + x^3}{x^2 + 2x - 3}$$



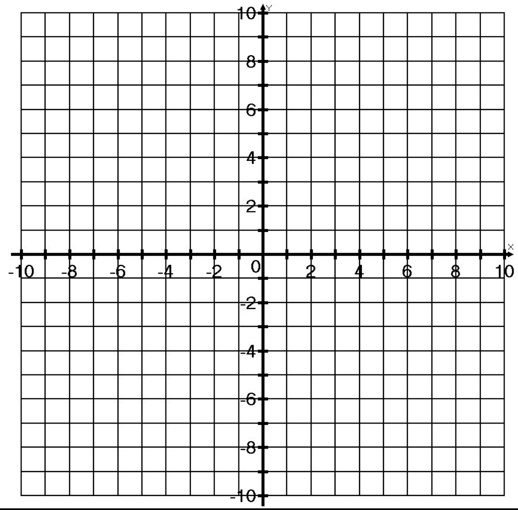
$$f(x)=\frac{x+1}{x^2+3x-10}$$

V.A.:

H.A.:

S.A.:

Hole:



D: **R:**

X-Int: **Y-Int:**

Inc: **Dec:**

End Behavior:

As x → -∞, f(x) → ___

As → ∞, f(x) → ___

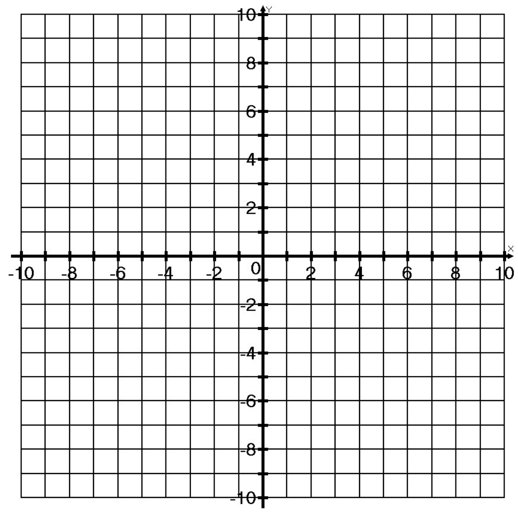
$$f(x)=\frac{5}{x^2-6x+8}$$

V.A.:

H.A.:

S.A.:

Hole:



D: **R:**

X-Int: **Y-Int:**

Inc: **Dec:**

End Behavior:

As x → -∞, f(x) → ___

As → ∞, f(x) → ___

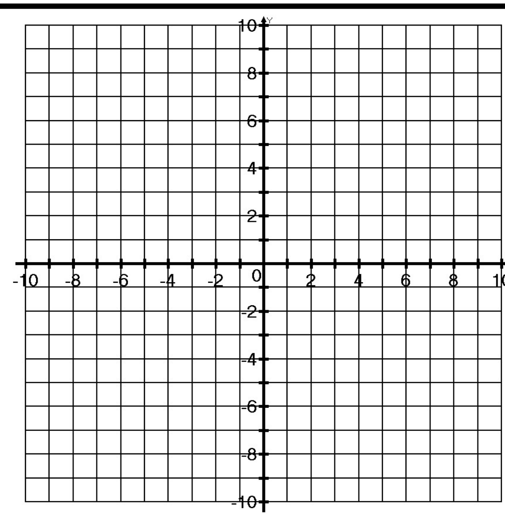
$$f(x)=\frac{2x^2+7x-4}{x^3-1}$$

V.A.:

H.A.:

S.A.:

Hole:



D: **R:**

X-Int: **Y-Int:**

Inc: **Dec:**

End Behavior:

As x → -∞, f(x) → ___

As → ∞, f(x) → ___