Algebra 2 Final Exam Review

Determine if the following symmetry is odd, even or neither. Then, factor (completely!) using any method.

1) $81x^4 - 1$ 2) $6x^5 + 15x^3 + 6x$ 3) $4x^6 - 5x^3 - 6$ 4) $6x^3 + 9x^2 + 2x + 3$

Solve the following equations and inequalities. Write inequality answers in interval notation.

5) $|2x - 5| = 7$ 6) $x^3 - 4x^2 + 7x + 40 = 3x^2 + 5x$ 7) $x + 5 = \sqrt{20x + 9}$

8) $(7x - 9)^{1/3} + 11 = 14$ 9) $-\frac{1}{3}|4 - 3x| = -6$ 10) $\sqrt{x + 56} + 3 = 19$

11) $2x^3 - 3x^2 - 17x + 30 = 0$ 12) $\frac{6}{5} = 1 + \frac{22}{x}$ 13) $\frac{2}{x - 3} = 4$

14) $x^4 + 27x = 0$ 15) $\frac{2}{x - 6} = \frac{-5}{x + 1}$ 16) $8^{3x-1} + 4 = 32$

Perform the following operations and simplify the expression.

17) $(3x^3 + 10x + 5) - (x^3 - 4x + 6)$ 18) $(2x - 3)(x + 7)(x + 6)$

19) $(2x^3 - 3x + 5) \div (x + 3)$ 20) $\frac{xy^8}{3x^{-2}} \cdot \frac{6x^3y^4}{2x^3y^{-2}}$

21) $(x^2 + x - 30) \div \frac{x^2 - 2x - 15}{x^2 + 7x + 12} \bullet \frac{x - 5}{x + 6}$ 22) $\frac{5}{x^2 - 9} - \frac{2 - x}{x^2 - 9}$

23) $\frac{10}{x^2 - 5x - 14} + \frac{1}{x - 7}$ 24) $(3x + 2)^4$

25) Expand: $\log_3 \frac{81x\sqrt[3]{y}}{2z^5}$ 26) Condense: $\ln 7 - 3\ln a + 4\ln b - \ln 14$

27) $\frac{x^2 - 4}{x^3 + 8}$ 28) Inverse of $y = 2x - 12$

29) List the possible rational roots for: $f(x) = 6x^4 + x^3 - 12x + 15$

30) Given $f(x) = -3x^3 + 7x^2 - 4x + 8$, what is $f(3)$?

31) Given $f(x) = 3x^2 - 2x + 8$ and $g(x) = 2x + 1$ what is $(f \circ g)x$?

32) Inverse of $y = 3\log_5(x + 2) - 7$

33) Given the zeros of a function are 0, -6, 3, & 5, if the leading coefficient is one, write the function of the polynomial.

34) Given (3x - 5) is a factor of a polynomial g(x), what else do you know?

35) What degree is the polynomial: $y = -3x^3(x+2)(x-4)^2$? Draw a SKETCH of the graph.

36) Given the sequence, what is the rule for the nth term? 2, 6, 18, 54, ...

37) What is the sum for the first 14 terms of 4 - 16 + 64 - 256 ...

Graph the following functions and give their characteristics. Also, for #38-43, ID the parent graph and describe the transformation.

38) $y = 2|x-3|-8$

39) $y = -3\sqrt{x+6}+4$

40) $y = \sqrt[3]{-4x}-1$

41) $y = \frac{-2}{x+3}-2$

42) $y = \frac{1}{2}\log_3(x+2)-3$

43) $y = -3\left(\frac{1}{2}\right)^{x+2}+1$

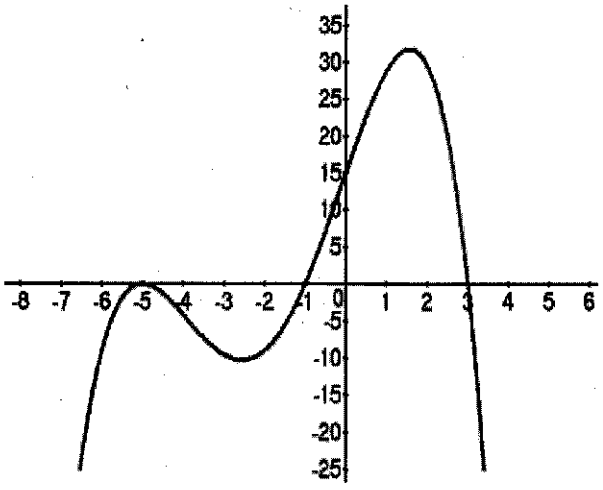
44) $f(x) = x^3+3x^2-4x-12$

45) $f(x) = \frac{x^2-16}{2x^2+13x+20}$ asymptotes? holes?

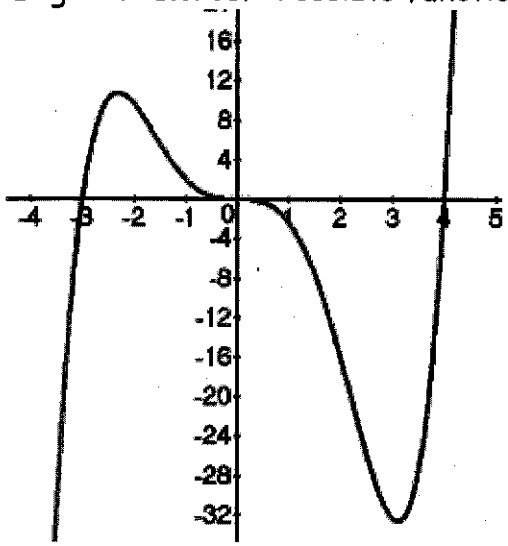
46) $f(x) = x^3-5x^2-18x+72$

47) What can you tell about the functions? LC type? Degree? Zeros? Possible function?

a.

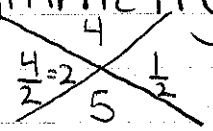


b.

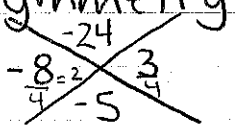


1. $81x^4 - 1$ even symmetry
$(9x^2-1)(9x^2+1)$
$(3x-1)(3x+1)(9x^2+1)$

2. $6x^5+15x^3+6x$ odd symmetry
$3x(2x^4+5x^2+2)$
$3x(2x+1)(x+2)$



3. $4x^6-5x^3-6$ neither symmetry
$(x^3-2)(4x^3+3)$



4. $(6x^3+9x^2)+(2x+3)$ neither symmetry
$3x^2(2x+3)+1(2x+3)$
$(3x^2+1)(2x+3)$

5. $|2x-5|=7$

$2x-5=7$ $\quad$ $2x-5=-7$
$2x=12$ $\quad$ $2x=-2$
$\boxed{x=6}$ $\quad$ $\boxed{x=-1}$

6. $x^3-4x^2+7x+40=3x^2+5x$
$-3x^2-5x$
$x^3-7x^2+2x+40=0$

$c=40$: $1,2,4,5,8,10,20,40$ $\quad$ $d=1$: 1

$\pm\{1,2,4,5,8,10,20,40\}$

-2	1	-7	2	40
		-2	18	-40
	1	-9	20	0

$(x+2)(x^2-9x+20)=0$
$(x+2)(x-4)(x-5)=0$

$\boxed{x=-2,\ 4,\ 5}$

7. $(x+5)^2 = (\sqrt{20x+9})^2$

$x^2+10x+25 = 20x+9$

$-20x-9 \quad -20x-9$

$x^2-10x+16 = 0$

$(x-8)(x-2) = 0$

$\boxed{x=8 \quad x=2}$

$8+5 = \sqrt{20(8)+9}$

$13 = \sqrt{169}$

$2+5 = \sqrt{20(2)+9}$

$7 = \sqrt{49}$

8. $(7x-9)^{1/3}+11 = 14$

$((7x-9)^{1/3})^3 = (3)^3$

$7x-9 = 27$

$7x = 36$

$\boxed{x = 36/7}$

9. $(-\frac{1}{3}|4-3x| = -6) \cdot -\frac{3}{1}$

$|4-3x| = 18$

$4-3x = 18 \qquad 4-3x = -18$

$-3x = 14 \qquad -3x = -22$

$\boxed{x = -14/3 \qquad x = 22/3}$

10. $\sqrt{x+56}+3 = 19$

$(\sqrt{x+56})^2 = (16)^2$

$x+56 = 256$

$\boxed{x = 200}$

11. $2x^3-3x^2-17x+30 = 0$

$c = 30$: $1, 2, 3, 5, 6, 10, 15, 30$

$d = 2$: $1, 2$

$\pm\{1, \frac{1}{2}, 2, 3, \frac{3}{2}, 5, \frac{5}{2}, 6, 10, 15, \frac{15}{2}, 30\}$

-30: $\frac{6}{2}=3$, $-\frac{5}{2}$, 1

$2 \mid 2 \quad -3 \quad -17 \quad 30$

$\qquad 4 \quad 2 \quad -30$

$2 \quad 1 \quad -15 \quad \boxed{0}$

$(x-2)(2x^2+x-15) = 0$

$(x-2)(x+3)(2x-5) = 0$

$\boxed{x = 2, -3, 5/2}$

12. $\left(\frac{6}{5} = 1 + \frac{22}{x}\right)(5x)$

$\frac{6(5x)}{5} = 1(5x) + \frac{22(5x)}{x}$

$6x = 5x + 110$

$\boxed{x = 110}$

13. $\frac{2}{x-3} = 4$

$2 = 4(x-3)$

$2 = 4x - 12$

$14 = 4x$

$\boxed{x = 14/4 = 7/2 \text{ or } 3.5}$

14. $x^4 + 27x = 0$

$x(x^3 + 27) = 0$

$x(x+3)(x^2 - 3x + 9) = 0$

$\boxed{x = 0}$ $\boxed{x = -3}$

$(-3)^2 - 4(1)(9) = -27$

$x = \frac{3 \pm \sqrt{-27}}{2}$

$\boxed{x = \frac{3 \pm 3i\sqrt{3}}{2}}$

15. $\frac{2}{x-6} = \frac{-5}{x+1}$ $x \neq 6, -1$

$2(x+1) = -5(x-6)$

$2x + 2 = -5x + 30$

$7x + 2 = 30$

$7x = 28$

$\boxed{x = 4}$

$\frac{2}{4-6} = \frac{-5}{4+1}$

$\frac{2}{-2} \stackrel{\checkmark}{=} \frac{-5}{5}$

16. $8^{3x-1}+4=32$

$8^{3x-1}=28$

$3x-1=\log_8 28$

$3x=\log_8 28+1$

$\boxed{x=\frac{1}{3}(\log_8 28+1)}$

17. $(3x^3+10x+5)-(x^3-4x+6)$

$3x^3+10x+5-x^3+4x-6$

$\boxed{2x^3+14x-1}$

18. $(2x-3)(x+7)(x+6)$

$(2x-3)(x^2+13x+42)$

	x^2	$13x$	42
$2x$	$2x^3$	$26x^2$	$84x$
-3	$-3x^2$	$-39x$	-126

$=\boxed{2x^3+23x^2+45x-126}$

19. $(2x^3-3x+5)\div(x+3)$

-3 | 2 0 -3 5

-6 18 -45

2 -6 15 | -40

$=\boxed{2x^2-6x+15+\frac{-40}{x+3}}$

20. $\frac{xy^8}{3x^{-2}}\cdot\frac{6x^3y^4}{2x^3y^{-2}}=\frac{xy^8\cdot x^2}{3}\cdot\frac{6x^3y^4\cdot y^2}{2x^3}$

$=\frac{6x^6y^{14}}{6x^3}=\boxed{x^3y^{14}}$

21. $(x^2+x-30)\div\frac{x^2-2x-15}{x^2+7x+12}\cdot\frac{x-5}{x+6}$

$=(x^2+x-30)\cdot\frac{x^2+7x+12}{x^2-2x-15}\cdot\frac{x-5}{x+6}=(x+6)(x-5)\cdot\frac{(x+3)(x+4)}{(x-5)(x+3)}\cdot\frac{(x-5)}{(x+6)}$

$=\boxed{(x-5)(x+4)\text{ or }x^2-x-20}$

22. $\frac{5}{x^2-9} - \frac{2-x}{x^2-9} = \frac{5-2+x}{x^2-9} = \frac{x+3}{x^2-9} = \frac{x+3}{(x+3)(x-3)} = \boxed{\frac{1}{x-3}}$

23. $\frac{10}{x^2-5x-14} + \frac{1}{x-7} = \frac{10}{(x-7)(x+2)} + \frac{1}{(x-7)} \cdot \frac{(x+2)}{(x+2)}$

$= \frac{10+1(x+2)}{(x-7)(x+2)} = \boxed{\frac{x+12}{(x-7)(x+2)}}$

24. $(3x+2)^4$

1	4	6	4	1
$(3x)^4$	$(3x)^3$	$(3x)^2$	$(3x)^1$	—
—	$(2)^1$	$(2)^2$	$(2)^3$	$(2)^4$

$\boxed{81x^4 + 216x^3 + 216x^2 + 96x + 16}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

25. Expand $\log_3 \frac{81x\sqrt[3]{y}}{27z^5} = \log_3 \frac{3^4 \cdot x \cdot y^{1/3}}{2 \cdot z^5}$

$= \log_3 3^4 + \log_3 x + \log_3 y^{1/3} - \log_3 2 - \log_3 z^5$

$= \boxed{4 + \log_3 x + \frac{1}{3}\log_3 y - \log_3 2 - 5\log_3 z}$

26. $\ln 7 - 3\ln a + 4\ln b - \ln 14$

$= \ln 7 - \ln a^3 + \ln b^4 - \ln 14$

$= \ln\left(\frac{7 \cdot b^4}{a^3 \cdot 14}\right) = \boxed{\ln\left(\frac{b^4}{2a^3}\right)}$

27. $\frac{x^2-4}{x^3+8} = \frac{(x-2)(x+2)}{(x+2)(x^2-2x+4)} = \boxed{\frac{x-2}{x^2-2x+4}}$

28. Inverse of $y = 2x - 12$

$x = 2y - 12$

$x + 12 = 2y$

$\boxed{y = \frac{1}{2}x + 6}$

29. possible rational roots for $f(x) = 6x^4 + x^3 - 12x + 15$

$c = 15$: $1, 3, 5, 15$ $\quad$ $d = 6$: $1, 2, 3, 6$

$$\boxed{\pm\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 3, \frac{3}{2}, 5, \frac{5}{2}, \frac{5}{3}, \frac{5}{6}, 15, \frac{15}{2}\right\}}$$

30. $f(x) = -3x^3 + 7x^2 - 4x + 8$, $f(3)$?

direct

$f(3) = -3(3)^3 + 7(3)^2 - 4(3) + 8$

$= \boxed{-22}$

synthetic

3	-3	7	-4	8
		-9	-6	-30
	-3	-2	-10	-22 *

$= \boxed{-22}$

31. $f(x) = 3x^2 - 2x + 8$, $g(x) = 2x + 1$, $(f \circ g)(x)$? $[f(g(x))]$

$(f \circ g)(x) = 3(2x+1)^2 - 2(2x+1) + 8$

$= 3(4x^2 + 4x + 1) - 2(2x + 1) + 8$

$= 12x^2 + 12x + 3 - 4x - 2 + 8$

$= \boxed{12x^2 + 8x + 9}$

$(2x+1)^2 = (2x+1)(2x+1)$

$= 4x^2 + 2x + 2x + 1$

$= 4x^2 + 4x + 1$

32. Inverse of $y = 3\log_5(x+2) - 7$

$x = 3\log_5(y+2) - 7$

$x + 7 = 3\log_5(y+2)$

$\frac{1}{3}(x+7) = \log_5(y+2)$

$5^{1/3(x+7)} = y + 2$

$\boxed{y = 5^{1/3(x+7)} - 2}$

33. $x = 0, -6, 3, +5$

$y = 1x(x+6)(x-3)(x-5)$

$= (x^2+6x)(x^2-8x+15)$

	x^2	$-8x$	$+15$
x^2	x^4	$-8x^3$	$15x^2$
$+6x$	$6x^3$	$-48x^2$	$90x$

$= \boxed{x^4 - 2x^3 - 33x^2 + 90x}$

34. $(3x-5)$ is a factor

$x = 5/3$ is a solution / root

$f(5/3) = 0$

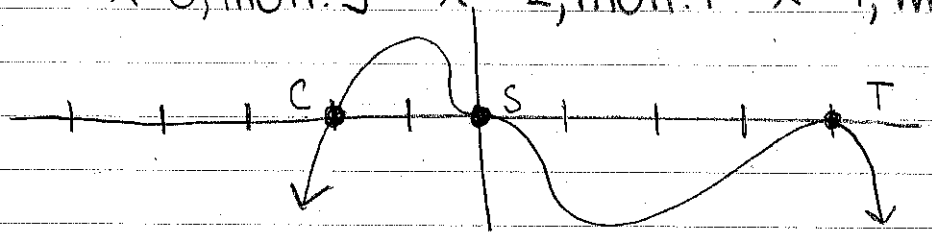
$(5/3, 0)$ is an x-intercept

there is no remainder when $\div (3x-5)$

35. $y = -3x^3(x+2)(x-4)^2$

degree: $3+1+2=6$ L.C. -3 Ends: ↙ ↘

$x=0$, mult. 3 $x=-2$, mult. 1 $x=4$, mult. 2



36. $2, 6, 18, 54, \ldots$

$a_1 = 2 \quad r = 3$ $\boxed{a_n = 2(3)^{n-1}}$

37. $4 - 16 + 64 - 256 \ldots$

$a_1 = 4 \quad r = -4$ $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$S_{14} = 4\left(\frac{1-(-4)^{14}}{1-(-4)}\right) = \boxed{-214,748,364}$

38. parent: $y = |x|$
· v stretch by 2, shift right 3 + down 8

39. parent: $y = \sqrt{x}$
· reflect over x-axis, v. stretch by 3, shift left 6 and up 4

40. parent: $y = \sqrt{x}$
· reflect over y-axis, h. compress by 1/4, shift down 1

41 parent: $y = \frac{1}{x}$
· reflect over x-axis, v. stretch by 2, shift left 3 and down 2

42. parent: $y = \log_3 x$
· v compress by $\frac{1}{2}$, shift left 2 and down 3

43. parent: $y = \frac{1}{2}^x$
· reflect over x-axis, v. stretch by 3, shift left 2 + up 1

47a. even degree, negative L.C.
$x = -5$, mult. 2 $x = -1$, mult. 1 $x = 3$, mult. 1

$f(x) = -\frac{1}{5}(x+5)^2(x+1)(x-3)$

b. odd degree, positive L.C.
$x = -3$, mult. 1 $x = 0$, mult. 3 $x = 4$, mult. 1

$f(x) = x^3(x+3)(x-4)$

Graphing Answer Sheet

38 Equation: $y = 2|x-3|-8$

Vertex: $(3,-8)$

Domain: $(-\infty, \infty)$

Range: $[-8, \infty)$

X-Int: $(-1,0)$ $(7,0)$

Y-Int: $(0,-2)$

Extrema: min @ $(3,-8)$

Int of Inc: $(3,\infty)$

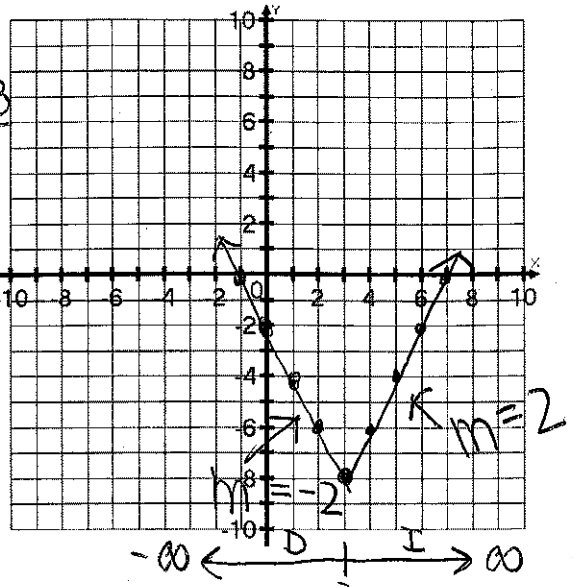
Int of Dec: $(-\infty,3)$

End Behavior: As $x \to -\infty$, $y \to \infty$

As $x \to \infty$, $y \to \infty$

x	y
-2	2
-1	1
0	0
1	1
2	2

x+3	2y-8
1	-4
2	-6
3	-8
4	-6
5	-4



39 Equation: $y = -3\sqrt{x+6}+4$

Vertex: $(-6,4)$

Domain: $[-6, \infty)$

Range: $(-\infty, 4]$

X-Int: $(-4.22, 0)$

Y-Int: $(0, -3\sqrt{6}+4)$ or $(0,-3.35)$

Extrema: max @ $(-6,4)$

Int of Inc: none

Int of Dec: $(-6,\infty)$

End Behavior: As $x \to \infty$, $y \to -\infty$

$0 = -3\sqrt{x+6}+4$

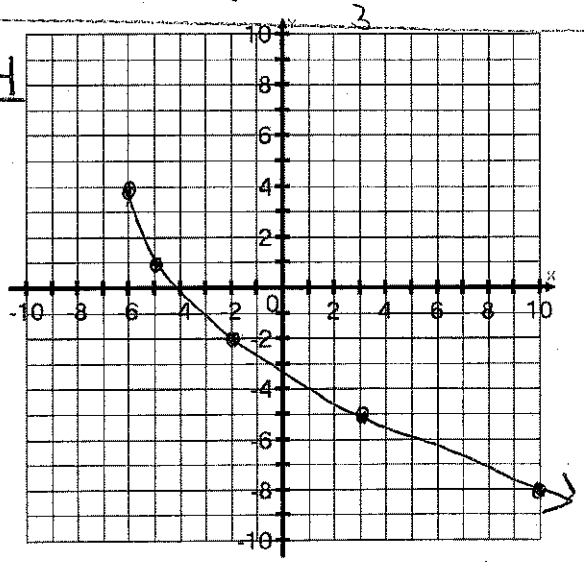
$\frac{4}{3} = \sqrt{x+6}$

$\frac{16}{9} = x+6$

$x = -4.22$

x	y
0	0
1	1
4	2
9	3
16	4

x-6	-3y+4
-6	4
-5	1
-2	-2
3	-5
10	-8



40 Equation: $y = \sqrt[3]{-4x} - 1$

Vertex: $(0,-1)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

X-Int: $(-1/4, 0)$

Y-Int: $(0,-1)$

Extrema: none

Int of Inc: none

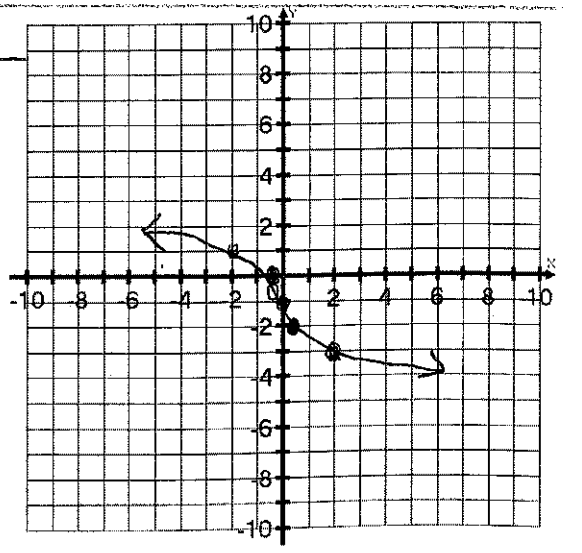
Int of Dec: $(-\infty,\infty)$

End Behavior: As $x \to -\infty$, $y \to \infty$

As $x \to \infty$, $y \to -\infty$

x	y
-8	-2
-1	-1
0	0
1	1
8	2

$-\frac{1}{4}x$	y-1
2	-3
$\frac{1}{4}$	-2
0	-1
$-\frac{1}{4}$	0
-2	1



Graphing Answer Sheet

(41) Equation: $y = \frac{-2}{x+3} - 2$

~~Vertex~~ VA: $x = -3$ H.A. $y = -2$

$0 = \frac{-2}{x+3} - 2$

Domain: $\{x \mid x \neq -3\}$

Range: $\{y \mid y \neq -2\}$

$2 = \frac{-2}{x+3}$

X-Int: $(-4, 0)$

Y-Int: $(0, -2.67)$

$\frac{1}{2} = \frac{x+3}{-2}$

Extrema: none

Int of Inc: $(-\infty, -3) \cup (-3, \infty)$

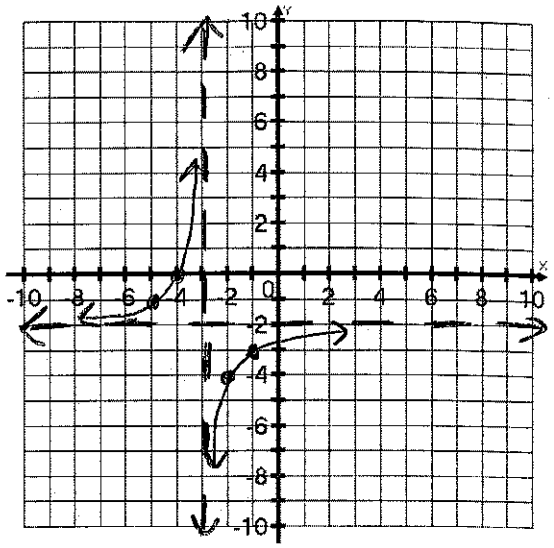
$-1 = x+3$

Int of Dec: none

End Behavior: As $x \to -\infty, y \to -2$

As $x \to \infty, y \to -2$

-5	-1
-4	0
-3	Und.
-2	-4
-1	-3



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(42) Equation: $y = \frac{1}{2}\log_3(x+2) - 3$

~~Vertex~~ VA: $x = -2$

$0 = \frac{1}{2}\log_3(x+2) - 3$

Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

$3 = \frac{1}{2}\log_3(x+2)$

X-Int: $(727, 0)$

$6 = \log_3(x+2)$

Y-Int: $(0, -2.68)$

$3^6 = x+2$

Extrema: none

$729 = x+2$

Int of Inc: $(-2, \infty)$

$727 = x$

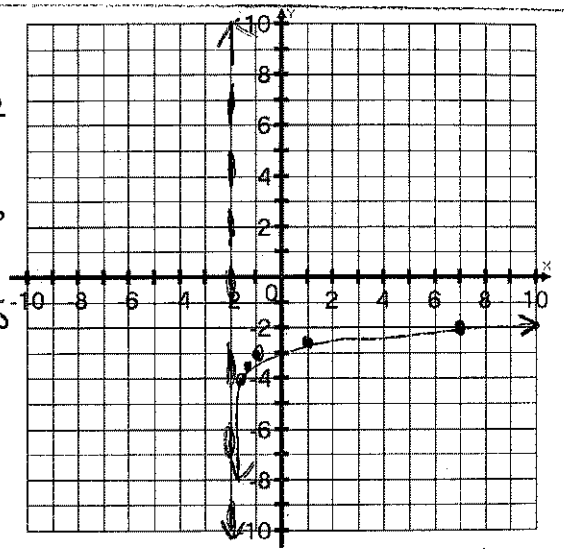
Int of Dec: none

End Behavior: As $x \to -2, y \to -\infty$

As $x \to \infty, y \to \infty$

exp x	exp 3^x	log	
-2	$\frac{1}{9}$	$\frac{1}{9}$	-2
-1	$\frac{1}{3}$	$\frac{1}{3}$	-1
0	1	1	0
1	3	3	1
2	9	9	2

$x-2$	$\frac{1}{2}y-3$
$-1\ 8/9$	-4
$-1\ 2/3$	-3.5
-1	-3
1	-2.5
7	-2



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(43) Equation: $y = -3\left(\frac{1}{2}\right)^{x+2} + 1$

~~Vertex~~ HA: $y = 1$

$0 = -3\left(\frac{1}{2}\right)^{x+2} + 1$

Domain: $(-\infty, \infty)$

$\frac{1}{3} = \frac{1}{2}^{x+2}$

Range: $(-\infty, 1)$

$\log_{\frac{1}{2}}\frac{1}{3} = x+2$

X-Int: $(-.42, 0)$

Y-Int: $(0, \frac{1}{4})$

$x = \log_{\frac{1}{2}}\frac{1}{3} - 2$

Extrema: none

$= -.42$

Int of Inc: $(-\infty, \infty)$

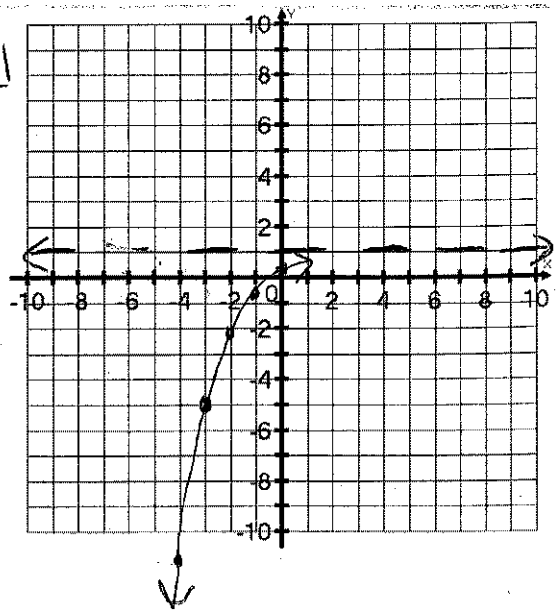
Int of Dec: none

End Behavior: As $x \to -\infty, y \to -\infty$

As $x \to \infty, y \to 1$

x	$\frac{1}{2}^x$
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

$x-2$	$-3y+1$
-4	-11
-3	-5
-2	-2
-1	$-\frac{1}{2}$
0	$\frac{1}{4}$



Graphing Answer Sheet

(44) Equation: $f(x) = x^3 + 3x^2 - 4x - 12$

$x^2(x+3) - 4(x+3)$

$(x^2-4)(x+3)$

$(x-2)(x+2)(x+3)$

$x = 2$ C

$x = -2$ C

$x = -3$ C

$-\infty$ I -2.53 D 0.53 I ∞

Vertex:

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

X-Int: $(2,0)$ $(-2,0)$ $(-3,0)$

Y-Int: $(0,-12)$

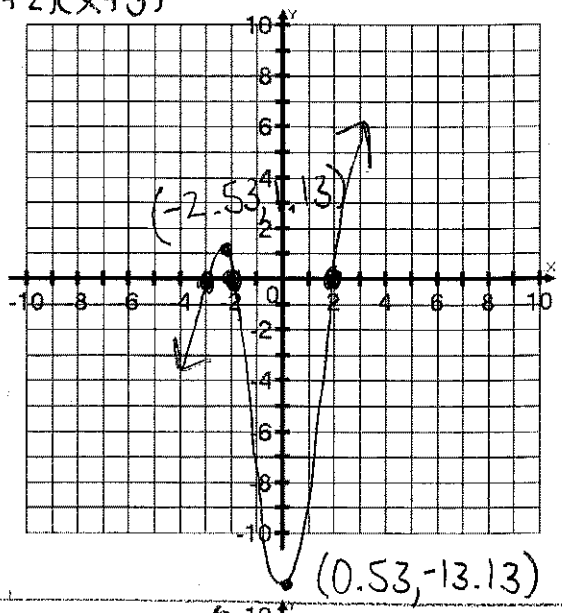
Extrema: max @ $(-2.53, 1.13)$, min @ $(0.53, -13.13)$

Int of Inc: $(-\infty, -2.53) \cup (0.53, \infty)$

Int of Dec: $(-2.53, 0.53)$

End Behavior: As $x \to -\infty$, $y \to -\infty$

As $x \to \infty$, $y \to \infty$



(45) Equation: $f(x) = \frac{x^2-16}{2x^2+13x+20} = \frac{(x-4)(x+4)}{(2x+5)(x+4)} = \frac{x-4}{2x+5}$

Vertex:

Domain: $\{x \mid x \neq -\frac{5}{2}, -4\}$ $x \neq -\frac{5}{2}, -4$ VA hole

Range:

X-Int: $(4,0)$

Y-Int: $(0, -\frac{4}{5})$

Extrema:

Int of Inc:

Int of Dec:

VA: $x = -\frac{5}{2}$

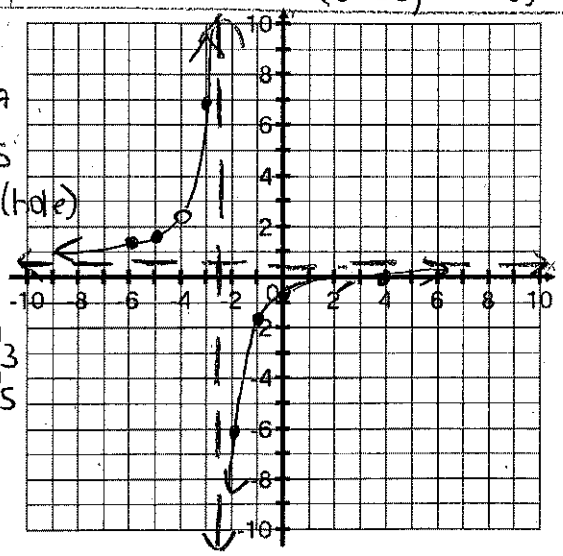
hole: $(-4, \frac{8}{3})$

HA: $y = \frac{1}{2}$

-6	$10/7$
-5	$9/5$
-4	$8/3$ (hole)
-3	7
-2	-6
-1	$-5/3$
0	$-4/5$

End Behavior: As $x \to -\infty$, $y \to \frac{1}{2}$

As $x \to \infty$, $y \to \frac{1}{2}$



(46) Equation: $f(x) = x^3 - 5x^2 - 18x + 72 = (x+4)(x-3)(x-6)$

Vertex:

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

X-Int: $(-4,0)(3,0)(6,0)$

Y-Int: $(0,72)$

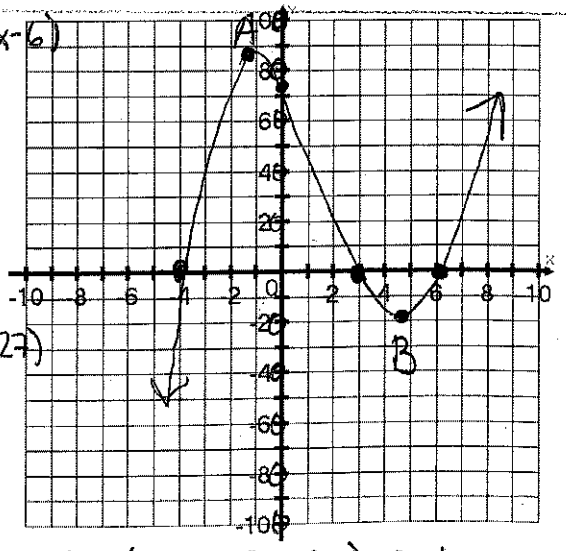
Extrema: max @ $(-1.30, 84.75)$ min @ $(4.63, -19.27)$

Int of Inc: $(-\infty, -1.30) \cup (4.63, \infty)$

Int of Dec: $(-1.30, 4.63)$

End Behavior: As $x \to -\infty$, $y \to -\infty$

As $x \to \infty$, $y \to \infty$



A = $(-1.30, 84.75)$ B = $(4.63, -19.27)$

$-\infty$ I -1.30 D 4.63 I ∞