Honors Algebra II

Unit 4B Review

Name ______________________________

Part 1: Solving by Factoring – factor completely and find all zeros

1. $p^3 - 4p^2 + 4p = 0$
2. $81w^4 - 16 = 0$
3. $64x^6 - 27x^3 = 0$
4. $4x^4 - 32x^2 + 64 = 0$
5. $4x^3 - 14x^2 + 6x - 21 = 0$
6. $x^4 - 11x^2 + 18 = 0$

Part 2: Evaluate the polynomial for the given value of *x* using synthetic substitution.

7. $f(x) = -2x^3 + 8x^2 + 3x - 1; x = 4$
8. $f(x) = x^4 + 2x^2 - 16; x = -3$
9. $4x^3 - 9x + 7; x = -5$

Part 3: List all possible rational roots and eliminate any duplicates.

10. $f(x) = 6x^3 - 8x^2 + 3x - 15$
11. $f(x) = -3x^3 + 20x^2 - 36x + 16$
12. $f(x) = 2x^4 - 11x^3 - 6x^2 + 42$

Part 4: Solve the following

13. $x^3 - 5x^2 + 30x - 10 = 6x + 10$
14. $12x^4 - 28x^3 + 7x = 4 - 13x^2$
15. $8x^7 - 56x^6 + 96x^5 = 0$

Part 5: a) Use the Rational Root Theorem and list all possible rational roots.

b) Use Descartes' Rule to determine all possible combinations of roots.

c) Algebraically, find all roots of the polynomial

16. $g(x) = x^3 + 6x^2 - 31x - 36$
17. $h(x) = 2x^4 + 5x^3 - 6x^2 - 7x + 6$
18. $m(x) = x^6 - 5x^4 + 2x^2 + 8$
19. $f(x) = 4x^5 + 8x^4 - 15x^3 - 23x^2 + 11x + 15$
20. $g(x) = x^{10} - x^8 + x^6 - x^4 + x^2 - 1$

Part 6: Verify that the given factor contains a zero of the polynomial. Then find the remaining factors.

21. $h(x) = x^4 - x^3 + 6x^2 + 14x - 20; \left(x - (1 - 3i)\right)$
22. $f(x) = 3x^4 + 4x^3 - 25x^2 + 26x - 8; (3x - 2)$
23. $g(x) = x^4 - 4x^3 + 8x - 32; (x - 4)$
24. $g(x) = 2x^4 - x^3 - 15x^2 + 6x + 18; \left(x - \sqrt{6}\right)$

Part 7: Writing Polynomials Given Zeros

25. The zeros of a polynomial, *P*(*x*), with real coefficients include 3, with a multiplicity of 2, and 3 + 4*i*. In addition, $P(0) = 675$. Write *P*(*x*) of minimum degree in factored form and standard form.

26. Write a polynomial function, f(x), of least degree in factored form and standard form that has rational coefficients, a leading coefficient of 1 and the given roots:

a. $3, 1 + \sqrt{2}$

b. $-4, 1, 1 - 2i$

Part 8: Solve the following polynomial inequalities. Graph the solutions on a number line and give solutions in interval notation.

27. $8x > 6x^2 - x^3$
28. $24x^2 \le x^4 + 2x^3$
29. $x^4 - 2x^2 > 63$
30. $2x^3 + 3x^2 < 29x - 30$

Part 9: Write a polynomial to solve the following word problem.

31. Find the dimensions for a rectangular solid whose height is 5 more than the width and whose length is 2 less than the width if the volume is 420ft3.

1. $p^3-4p^2+4p=0$

$p(p^2-4p+4)=0$

$p(p-2)^2=0$

$$\boxed{\underset{M.1}{p=0},\ \underset{M.2}{p=2}}$$

2. $81w^4-16=0$

$(9w^2-4)(9w^2+4)=0$

$(3w-2)(3w+2)(9w^2+4)=0$

$w^2=-\frac{9}{4}$

$w=\pm\frac{3i}{2}$

$$\boxed{\underset{M.1}{w=\frac{2}{3}},\ \underset{M.1}{w=-\frac{2}{3}},\ \underset{M.1}{w=\pm\frac{3i}{2}}}$$

3. $64x^6-27x^3=0$

$x^3(64x^3-27)=0$

$x^3(4x-3)(16x^2+12x+9)=0$

$(12)^2-4(16)(9)=-432$

$x=\frac{-12\pm\sqrt{-432}}{2(16)}=\frac{-12\pm 12i\sqrt{3}}{32}=-\frac{3}{8}\pm\frac{3i\sqrt{3}}{8}$

$$\boxed{\underset{M.3}{x=0},\ \underset{M.1}{x=\frac{3}{4}},\ \underset{M.1}{x=-\frac{3}{8}\pm\frac{3i\sqrt{3}}{8}}}$$

4. $4x^4-32x^2+64=0$

$4(x^4-8x^2+16)=0$

$4(x^2-4)^2=0$

$4(x-2)^2(x+2)^2=0$

$$\boxed{\underset{M.2}{x=2},\ \underset{M.2}{x=-2}}$$

5 $(4x^3-14x^2)+(6x-21)=0$

$2x^2(2x-7)+3(2x-7)=0$

$(2x^2+3)(2x-7)=0$

$2x^2+3=0$

$x^2=-\frac{3}{2}$

$x=\pm\frac{i\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\pm\frac{i\sqrt{6}}{2}$

$$\boxed{\underset{M.1}{x=\frac{7}{2}},\ \underset{M.1}{x=\pm\frac{i\sqrt{6}}{2}}}$$

6. $x^4-11x^2+18=0$

$(x^2-9)(x^2-2)=0$

$(x-3)(x+3)(x^2-2)=0$

$\boxed{\begin{array}{ccc} x=3, & x=-3, & x=\pm\sqrt{2} \\ \text{M.1} & \text{M.1} & \text{M.1} \end{array}}$

7. $f(x)=-2x^3+8x^2+3x-1, \quad x=4$

$$\begin{array}{r|rrrr} 4 & -2 & 8 & 3 & -1 \\ & & -8 & 0 & 12 \\ \hline & -2 & 0 & 3 & \boxed{11} \end{array}$$

$\boxed{f(4)=11}$

8. $f(x)=x^4+2x^2-16, \quad x=-3$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 2 & 0 & -16 \\ & & -3 & 9 & -33 & 99 \\ \hline & 1 & -3 & 11 & -33 & \boxed{83} \end{array}$$

$\boxed{f(-3)=83}$

9. $4x^3-9x+7, \quad x=-5$

$$\begin{array}{r|rrrr} -5 & 4 & 0 & -9 & 7 \\ & & -20 & 100 & -455 \\ \hline & 4 & -20 & 91 & \boxed{-448} \end{array}$$

$\boxed{y=-448 \text{ when } x=-5}$

10. $f(x)=6x^3-8x^2+3x-15$

$\underline{p=15}$

1, 3, 5, 15

$\underline{q=6}$

1, 2, 3, 6

$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{3}{1}, \frac{3}{2}, \cancel{\frac{3}{3}}, \cancel{\frac{3}{6}}, \frac{5}{1}, \frac{5}{2}, \frac{5}{3}, \frac{5}{6}, \frac{15}{1}, \frac{15}{2}, \cancel{\frac{15}{3}}, \cancel{\frac{15}{6}}$

$\boxed{\pm1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm3, \pm\frac{3}{2}, \pm5, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}, \pm15, \pm\frac{15}{2}}$

11. $f(x)=-3x^3+20x^2-36x+16$

$\underline{p=16}$

1, 2, 4, 8, 16

$\underline{q=3}$

1, 3

$\boxed{\pm1, \pm\frac{1}{3}, \pm2, \pm\frac{2}{3}, \pm4, \pm\frac{4}{3}, \pm8, \pm\frac{8}{3}, \pm16, \pm\frac{16}{3}}$

12. $f(x) = 2x^4 - 11x^3 - 6x^2 + 42$

$\underline{p = 42}$

1, 2, 3, 6, 7, 14, 21, 42

$\underline{q = 2}$

1, 2

$$\boxed{\pm 1, \pm \tfrac{1}{2}, \pm 2, \pm 3, \pm \tfrac{3}{2}, \pm 6, \pm 7, \pm \tfrac{7}{2}, \pm 14, \pm 21, \pm \tfrac{21}{2}, \pm 42}$$

13. $x^3 - 5x^2 + 30x - 10 = 6x + 10$

$\qquad\qquad -6x - 10$

$x^3 - 5x^2 + 24x - 20 = 0$

$\pm 1, \pm 2, \pm 4, \pm 10, \pm 20$

+	3	1
−	0	0
i	0	2

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 24 & -20 \\ & & 1 & -4 & 20 \\ \hline & 1 & -4 & 20 & \boxed{0} \end{array}$$

$(x-1)(x^2 - 4x + 20)$

$\boxed{\begin{array}{c} x = 1 \\ M.1 \end{array}}$ $\quad x = \dfrac{4 \pm \sqrt{-64}}{2} = \dfrac{4 \pm 8i}{2} = \boxed{2 \pm 4i \quad M.1}$

14. $12x^4 - 28x^3 + 7x = 4 - 13x^2$

$12x^4 - 28x^3 + 13x^2 + 7x - 4 = 0$

possible

roots: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \pm \frac{1}{6}, \pm \frac{1}{12}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}$

+	2	0	2	0
−	2	2	0	0
i	0	2	2	4

$$\begin{array}{r|rrrrr} 1 & 12 & -28 & 13 & 7 & -4 \\ & & 12 & -16 & -3 & 4 \\ \hline & 12 & -16 & -3 & 4 & \boxed{0} \end{array}$$

$2 \,|\, 12$

$$\boxed{\begin{array}{l} x = 1 \quad M.1 \\ x = \frac{1}{2} \quad M.1 \\ x = -\frac{1}{2} \quad M.1 \\ x = \frac{4}{3} \quad M.1 \end{array}}$$

$(x-1)(12x^3 - 16x^2 - 3x + 4) = 0$

$(x-1)(4x^2 - 1)(3x - 4) = 0$

$(x-1)(2x-1)(2x+1)(3x-4) = 0$

15. $8x^7-56x^6+96x^5=0$

$8x^5(x^2-7x+12)=0$

$8x^5(x-3)(x-4)=0$

$$\boxed{\begin{array}{ccc} x=0, & x=3, & x=4 \\ \text{M.5} & \text{M.1} & \text{M.1} \end{array}}$$

16. $g(x)=x^3+6x^2-31x-36$

a. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

b. $g(x)=x^3\underbrace{+6}_{x}x^2\underbrace{-31}_{1}x\underbrace{-36}_{x}$; $g(-x)=-x^3\underbrace{+6}_{1}x^2\underbrace{+31}_{x}x\underbrace{-36}_{2}$

Max positive: 1

Max negative: 2

pos	1	1
neg	2	0
imag	0	2

c.
$$\begin{array}{r|rrrr} -1 & 1 & 6 & -31 & -36 \\ & & -1 & -5 & 36 \\ \hline & 1 & 5 & -36 & \boxed{0} \end{array}$$

$(x+1)(x^2+5x-36)=0$

$(x+1)(x+9)(x-4)=0$

$$\boxed{\begin{array}{ccc} x=-1, & x=-9, & x=4 \\ \text{M.1} & \text{M.1} & \text{M.1} \end{array}}$$

17. $h(x)=2x^4+5x^3-6x^2-7x+6$

a. $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

b. $h(x)=2x^4\underbrace{+5}_{x}x^3\underbrace{-6}_{1}x^2\underbrace{-7}_{x}x\underbrace{+6}_{2}$; $h(-x)=2x^4\underbrace{-5}_{1}x^3\underbrace{-6}_{x}x^2\underbrace{+7}_{2}x\underbrace{+6}_{x}$

Max positive: 2

Max negative: 2

pos	2	2	0	0
neg	2	0	2	0
imag	0	2	2	4

$$\begin{array}{r|rrrrr} 1 & 2 & 5 & -6 & -7 & 6 \\ & & 2 & 7 & 1 & -6 \\ \hline \cancel{1} & 2 & 7 & 1 & -6 & \boxed{0} \\ & & \cancel{2} & \cancel{9} & \cancel{10} & \\ \hline & \cancel{2} & \cancel{9} & \cancel{10} & \cancel{4} & \end{array}$$

$$\begin{array}{r|rrrr} -3 & 2 & 7 & 1 & -6 \\ & & -6 & -3 & 6 \\ \hline & 2 & 1 & -2 & \boxed{0} \end{array}$$

$(x-1)(x+3)(2x^2+x-2)$

$$\boxed{\begin{array}{ccc} x=1 & x=-3 & x=\dfrac{-1\pm\sqrt{17}}{4} \\ \text{M.1} & \text{M.1} & \text{M.1} \end{array}}$$

18. $m(x) = x^6 - 5x^4 + 2x^2 + 8$

a. ±1, ±2, ±4, ±8

b. $m(x) = x^6 \underbrace{-5x^4}_{1} \underbrace{+2x^2}_{2} \underbrace{+8}_{x}$ $\quad m(-x) = x^6 \underbrace{-5x^4}_{1} \underbrace{+2x^2}_{2} \underbrace{+8}_{x}$

Max positive: 2

Max negative: 2

pos	2	2	0	0
neg	2	0	2	0
imag	2	4	4	6

$$\begin{array}{r|rrrrrrr}
2 & 1 & 0 & -5 & 0 & 2 & 0 & 8 \\
 & & 2 & 4 & -2 & -4 & -4 & -8 \\
\hline
2 & 1 & 2 & -1 & -2 & -2 & -4 & \boxed{0} \\
 & & \cancel{2} & \cancel{8} & \cancel{14} & & & \\
\hline
 & \cancel{1} & \cancel{4} & \cancel{7} & \cancel{12} & & &
\end{array}$$

$$\begin{array}{r|rrrrrr}
-2 & 1 & 2 & -1 & -2 & -2 & -4 \\
 & & -2 & 0 & 2 & 0 & 4 \\
\hline
 & 1 & 0 & -1 & 0 & -2 & \boxed{0}
\end{array}$$

b/c even symmetry

$(x+2)(x-2)(x^4-x^2-2) = 0$

$(x+2)(x-2)(x^2-2)(x^2+1) = 0$

$$\boxed{\begin{array}{cccc} x=-2 & x=2 & x=\pm\sqrt{2} & x=\pm i \\ \text{M.1} & \text{M.1} & \text{M.1} & \text{M.1} \end{array}}$$

19. $f(x) = 4x^5 + 8x^4 - 15x^3 - 23x^2 + 11x + 15$

a. $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm 5, \pm\frac{5}{2}, \pm\frac{5}{4}, \pm 15, \pm\frac{15}{2}, \pm\frac{15}{4}$

b. $f(x) = 4x^5 \underbrace{+8x^4}_{x} \underbrace{-15x^3}_{1} \underbrace{-23x^2}_{x} \underbrace{+11x}_{2} \underbrace{+15}_{x}$ $\quad$ Max pos 2

$f(-x) = -4x^5 \underbrace{+8x^4}_{1} \underbrace{+15x^3}_{x} \underbrace{-23x^2}_{2} \underbrace{-11x}_{x} \underbrace{+15}_{3}$ $\quad$ Max neg 3

pos	2	0	2	0
neg	3	3	1	1
imag	0	2	2	4

$$\begin{array}{r|rrrrrr}
+1 & 4 & 8 & -15 & -23 & 11 & 15 \\
 & & 4 & 12 & -3 & -26 & -15 \\
\hline
-1 & 4 & 12 & -3 & -26 & -15 & \boxed{0} \\
 & & -4 & -8 & 11 & 15 & \\
\hline
-1 & 4 & 8 & -11 & -15 & \boxed{0} & \\
 & & -4 & -4 & 15 & & \\
\hline
 & 4 & 4 & -15 & \boxed{0} & &
\end{array}$$

$(x-1)(x+1)^2(4x^2+4x-15)=0$

$(x-1)(x+1)^2(4x^2+10x-6x-15)=0$

$(x-1)(x+1)^2(2x-3)(2x+5)=0$

$$\boxed{\begin{array}{cccc} x=1, & x=-1, & x=3/2, & x=-5/2 \\ M.1 & M.2 & M.1 & M.1 \end{array}}$$

20. $g(x)=x^{10}-x^{8}+x^{6}-x^{4}+x^{2}-1$

a. ± 1

b. $g(x)=x^{10}\underbrace{-}_{1}x^{8}\underbrace{+}_{2}x^{6}\underbrace{-}_{3}x^{4}\underbrace{+}_{4}x^{2}\underbrace{-}_{5}1$ Max pos
5

$g(-x)=x^{10}\underbrace{-}_{1}x^{8}\underbrace{+}_{2}x^{6}\underbrace{-}_{3}x^{4}\underbrace{+}_{4}x^{2}\underbrace{-}_{5}1$ Max neg
5

pos	5	5	5	3	3	3	1	1	1
neg	5	3	1	5	3	1	5	3	1
imag	0	2	4	2	4	6	4	6	8

$$\begin{array}{r|rrrrrrrrrrr}
1 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\
 & & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
\hline
-1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & \boxed{0} \\
 & & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & \\
\hline
 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & \boxed{0} &
\end{array}$$

$(x+1)(x-1)(x^{8}+x^{4}+1)=0$

$$\boxed{\begin{array}{cc} x=-1 & x=1 \\ M.1 & M.1 \end{array}}$$

↑ can't solve but includes 8 solutions

21. $h(x) = x^4 - x^3 + 6x^2 + 14x - 20$; $(x-(1-3i))$

Verify $x = 1-3i$ is a root

$$\begin{array}{r|rrrrr} 1-3i & 1 & -1 & 6 & 14 & -20 \\ & & 1-3i & -9-3i & -12+6i & 20 \\ \hline & 1 & -3i & -3-3i & 2+6i & \boxed{0} \end{array}$$

$-3i(1-3i) = -3i + 9i^2$

$(1-3i)(-3-3i) = -3-3i+9i+9i^2$

$= -3+6i-9$

$(1-3i)(2+6i) = 2+6i-6i-18i^2$

According to the remainder theorem $x = 1-3i$ is a root and $(x-(1-3i))$ is a factor

$$\begin{array}{r|rrrr} 1+3i & 1 & -3i & -3-3i & 2+6i \\ & & 1+3i & 1+3i & -2-6i \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$

$(x-1+3i)$

$(x-(1-3i))(x-(1+3i))(x^2+x-2)$

remaining factors: $(x-(1+3i))(x+2)(x-1)$

$(x-1-3i)$

22. $f(x) = 3x^4 + 4x^3 - 25x^2 + 26x - 8$; $(3x-2)$

Verify $x = 2/3$ is a root

$$\begin{array}{r|rrrrr} 2/3 & 3 & 4 & -25 & 26 & -8 \\ & & 2 & 4 & -14 & 8 \\ \hline & 3 & 6 & -21 & 12 & \boxed{0} \\ \hline & & & 3 & & \end{array}$$

$(3x-2)(x^3+2x^2-7x+4)$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -7 & 4 \\ & & 1 & 3 & -4 \\ \hline & 1 & 3 & -4 & \boxed{0} \end{array}$$

$(3x-2)(x-1)(x^2+3x-4) = 0$

remaining factors: $(x-1)^2(x+4)$

23. $g(x) = x^4 - 4x^3 + 8x - 32$; $(x-4)$

verify $x = 4$ is a root

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 8 & -32 \\ & & 4 & 0 & 0 & 32 \\ \hline & 1 & 0 & 0 & 8 & \boxed{0} \end{array}$$

↙ Yes, according to remainder theorem

$(x-4)(x^3+8)$

remaining factors: $(x+2)(x^2-2x+4)$

24. $g(x) = 2x^4 - x^3 - 15x^2 + 6x + 18$; $(x - \sqrt{6})$

verify $x = \sqrt{6}$ is a root

$$\begin{array}{r|rrrrr} \sqrt{6} & 2 & -1 & -15 & 6 & 18 \\ & & 2\sqrt{6} & 12-\sqrt{6} & -6-3\sqrt{6} & -18 \\ \hline -\sqrt{6} & 2 & -1+2\sqrt{6} & -3-\sqrt{6} & -3\sqrt{6} & \boxed{0} \\ & & -2\sqrt{6} & -\sqrt{6} & 3\sqrt{6} & \\ \hline & 2 & -1 & -3 & \boxed{0} & \end{array}$$

↙ Yes, according to remainder theorem

$(x-\sqrt{6})(x+\sqrt{6})(2x^2-x-3)$

remaining factors: $(x+\sqrt{6})(2x-3)(x+1)$

25. $P(x) = (x-3)^2(x-(3+4i))(x-(3-4i))$; $P(0) = 675$

$P(x) = (x-3)^2(x-3-4i)(x-3+4i)$

$= (x-3)^2(x^2-6x+9-16i^2)$

$= (x-3)^2(x^2-6x+25)$

$675 = a(-3)^2(25)$

$675 = 225a \quad \therefore a = 3$

$\boxed{P(x) = 3(x-3)^2(x^2-6x+25)}$ [FACTORED FORM]

$= 3(x^2-6x+9)(x^2-6x+25)$

$x^4 - 6x^3 + 25x^2$

$-6x^3 + 36x^2 - 150x \quad = x^4 - 12x^3 + 70x^2 - 204x + 225$

$9x^2 - 54x + 225$

$\boxed{P(x) = 3x^4 - 36x^3 + 210x^2 - 612x + 675}$ [STANDARD FORM]

26. $f(x) = (x-3)(x-(1+\sqrt{2}))(x-(1-\sqrt{2}))$

$= (x-3)(x-1-\sqrt{2})(x-1+\sqrt{2})$

$= (x-3)(x^2-2x+1-2)$

$\boxed{f(x) = (x-3)(x^2-2x-1)}$ [FACTORED FORM]

$= x^3-2x^2-x-3x^2+6x+3$

$\boxed{f(x) = x^3-5x^2+5x+3}$ [STANDARD FORM]

b. $f(x) = (x+4)(x-1)(x-(1-2i))(x-(1+2i))$

$= (x+4)(x-1)(x-1+2i)(x-1-2i)$

$= (x+4)(x-1)(x^2-2x+1-4i^2)$

$\boxed{f(x) = (x+4)(x-1)(x^2-2x+5)}$ [FACTORED FORM]

$= (x^2+3x-4)(x^2-2x+5)$

$x^4-2x^3+5x^2$

$3x^3-6x^2+15x$

$-4x^2+8x-20$

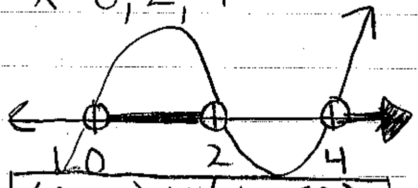
$\boxed{f(x) = x^4+x^3-5x^2+23x-20}$ [STANDARD FORM]

27. $8x > 6x^2-x^3$

$x^3-6x^2+8x > 0$

$x(x-4)(x-2) > 0$

$x = 0, 2, 4$



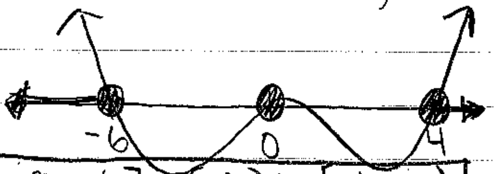
$\boxed{(0,2) \cup (4,\infty)}$

28. $24x^2 \leq x^4+2x^3$

$0 \leq x^4+2x^3-24x^2$

$0 \leq x^2(x+6)(x-4)$

$x = 0$ (M.2) $x = -6$, $x = 4$



$\boxed{(-\infty,-6] \cup \{0\} \cup [4,\infty)}$

29. $x^4-2x^2 > 63$

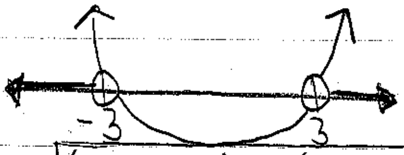
$x^4-2x^2-63 > 0$

$(x^2-9)(x^2+7) > 0$

$x = -3, 3, \pm i\sqrt{7}$

$\uparrow$

not important for inequalities



$\boxed{(-\infty,-3) \cup (3,\infty)}$

30. $2x^3+3x^2<29x-30$

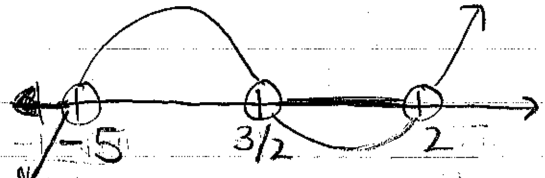
$2x^3+3x^2-29x+30<0$

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -29 & 30 \\ & & 4 & 14 & -30 \\ \hline & 2 & 7 & -15 & \boxed{0} \end{array}$$

$(x-2)(2x^2+7x-15)<0$

$(x-2)(2x-3)(x+5)<0$

$x=2, \frac{3}{2}, -5$



$\boxed{(-\infty,-5)\cup(\frac{3}{2},2)}$

31. Width = w height = w+5 length = w-2

$420 = w(w+5)(w-2)$

$0=w^3+3w^2-10w-420$

$$\begin{array}{r|rrrr} \cancel{3} & 1 & 3 & -10 & -420 \\ & & 3 & \cancel{18} & \cancel{24} \\ \hline & 1 & \cancel{6} & 8 & \end{array}$$

$$\begin{array}{r|rrrr} \cancel{4} & 1 & 3 & -10 & -420 \\ & & 4 & \cancel{28} & \cancel{72} \\ \hline & 1 & \cancel{7} & 18 & \end{array}$$

$$\begin{array}{r|rrrr} \cancel{5} & 1 & 3 & -10 & -420 \\ & & 5 & 40 & \cancel{150} \\ \hline & 1 & \cancel{8} & 30 & \end{array}$$

$$\begin{array}{r|rrrr} \cancel{6} & 1 & 3 & -10 & -420 \\ & & 6 & \cancel{54} & \\ \hline & 1 & \cancel{9} & 44 & \end{array}$$

$$\begin{array}{r|rrrr} 7 & 1 & 3 & -10 & -420 \\ & & 7 & 70 & 420 \\ \hline & 1 & 10 & 60 & \boxed{0} \end{array}$$

$w=7$

possible roots

~~1~~, ~~2~~, 3, 4, 5, 6, 7, 10, 12, 15, 20, 21, 28, 35, 42, 60, 70, 84, 105, 140, 210, 420

$w=7$

$h=7+5=12$

$l=7-2=5$

$\boxed{\text{The solid is } 7\text{ ft} \times 12\text{ ft} \times 5\text{ ft}}$