

# FTOA Practice #2

$$1. \quad x^4 - 3x^3 - 10x^2 + 24x = 0$$

$$x(x^3 - 3x^2 - 10x + 24) = 0$$

Max # +  
2

$$f(-x) = x^4 + 3x^3 - 10x^2 - 24x$$

$$f(-x) = x(x^3 + 3x^2 - 10x - 24)$$

Max # -  
1

+	2	0
-	1	1
i	0	2

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$x=0$

$$x(x-2)(x^2-x-12) = x(x-2)(x-4)(x+3)$$

$$\boxed{x=0 \quad x=2 \quad x=4 \quad x=-3}$$

$$2. \quad 15x^3 - 29x^2 + 17x - 3 = 0$$

Max # +  
3

$$f(-x) = -15x^3 - 29x^2 - 17x - 3$$

Max # -  
0

+	3	1
-	0	0
i	0	2

$$\begin{array}{r|rrrr} 1 & 15 & -29 & 17 & -3 \\ & & 15 & -14 & 3 \\ \hline & 15 & -14 & 3 & 0 \end{array}$$

$$(x-1)(15x^2 - 14x + 3) = 0$$

$$(x-1)(15x^2 - 9x - 5x + 3) = 0$$

$$(x-1)(3x-1)(5x-3) = 0$$

$$\boxed{x=1}$$

$$\boxed{x=\frac{1}{3}}$$

$$\boxed{x=\frac{3}{5}}$$

$$\begin{array}{r} 45 \\ -9 \times -5 \\ \hline -14 \end{array}$$

$$3. \quad x^4 - 18x^2 + 65 = 0$$

Max # +  
2

$$f(x) = x^4 - 18x^2 + 65$$

Max # -  
2

+	2	2	0	0
-	2	0	2	0
i	0	2	2	4

$$\begin{array}{r} \cancel{65} \\ -5 \quad -13 \\ \hline -18 \end{array}$$

$$(x^2 - 5)(x^2 - 13)$$

$$\boxed{x = \pm\sqrt{5} \quad x = \pm\sqrt{13}}$$

$$4. \quad x^3 - 2x^2 - 15x = 0$$

Max # +  
1

$$x(x^2 - 2x - 15) = 0$$

$$f(x) = -x^3 - 2x^2 + 15x$$

Max # -  
1

+	1
-	1
i	0

(other is  $x=0$ )

$$\boxed{\begin{array}{l} x=0 \\ x=5 \\ x=-3 \end{array}}$$

$$x(x-5)(x+3) = 0$$

$$5. \quad 16x^4 - 65x^2 + 4 = 0$$

Max # +  
2

$$f(x) = 16x^4 - 65x^2 + 4$$

Max # -  
2

+	2	2	0	0
-	2	0	2	0
i	0	2	2	4

$$\begin{array}{r} \cancel{64} \\ -64 \quad -1 \\ \hline -65 \end{array}$$

$$(16x^4 - 64x^2)(-1x^2 + 4) = 0$$

$$16x^2(x^2 - 4) - 1(x^2 - 4) = 0$$

$$(16x^2 - 1)(x^2 - 4) = 0$$

$$(4x-1)(4x+1)(x-2)(x+2) = 0$$

$$\boxed{\begin{array}{l} x = \pm \frac{1}{4} \\ x = \pm 2 \end{array}}$$

$$6. \quad 12x^3 - 8x^2 - x + 1 = 0$$

Max # +  
2

$$f(-x) = -12x^3 - 8x^2 + x + 1$$

Max # -  
1

$$\begin{array}{r|l} + & 2 & 0 \\ - & 1 & 1 \\ \hline i & 0 & 2 \end{array}$$

$$\frac{1}{2} \Big| \begin{array}{cccc} 12 & -8 & -1 & 1 \\ & 6 & -1 & -1 \\ \hline 12 & -2 & -2 & 0 \end{array}$$

$$\boxed{x = \frac{1}{2} \text{ M. 2.}} \\ \boxed{x = -\frac{1}{3}}$$

$$(2x-1)(6x^2-x-1) = 0 \\ (2x-1)(3x+1)(2x-1) = 0$$

$$7. \quad 36x^4 - 25x^2 + 4 = 0$$

Max # +  
2

$$f(-x) = 36x^4 - 25x^2 + 4$$

Max # -  
2

$$\begin{array}{r|l|l|l|l} + & 2 & 2 & 0 & 0 \\ - & 2 & 0 & 2 & 0 \\ \hline i & 0 & 2 & 2 & 4 \end{array}$$

$$\begin{array}{r} 144 \\ -9 \times -16 \\ -25 \end{array}$$

$$(36x^4 - 9x^2)(16x^2 + 4) = 0 \\ 9x^2(4x^2 - 1) - 4(4x^2 - 1) = 0$$

$$(9x^2 - 4)(4x^2 - 1) = 0$$

$$(3x-2)(3x+2)(2x-1)(2x+1) = 0$$

$$\boxed{x = \pm \frac{2}{3}} \quad \boxed{x = \pm \frac{1}{2}}$$

$$8. \quad x^3 + 6x^2 + 4x + 24 = 0$$

Max # +  
0

$$f(-x) = -x^3 + 6x^2 - 4x + 24$$

Max # -  
3

$$\begin{array}{r|l} + & 0 & 0 \\ - & 3 & 1 \\ \hline i & 0 & 2 \end{array}$$

$$(x^3 + 6x^2)(4x + 24) = 0$$

$$x^2(x+6) + 4(x+6) = 0$$

$$(x^2 + 4)(x+6) = 0$$

$$\boxed{x = \pm 2i, -6}$$

$$9. \quad x^4 - 3x^3 - 8x^2 + 22x - 24 = 0$$

$$\begin{array}{r|l} \text{Max \# +} & + 3 \quad | \quad 1 \\ \hline & - 1 \quad | \quad 1 \end{array}$$

$$f(-x) = x^4 + 3x^3 - 8x^2 - 22x - 24$$

$$\begin{array}{r|l} \text{Max \# -} & i \quad 0 \quad | \quad 2 \\ \hline & \end{array}$$

$$\begin{array}{r} 4 \mid 1 \quad -3 \quad -8 \quad 22 \quad -24 \\ \hline \end{array}$$

$$\begin{array}{r} \phantom{4 \mid} \phantom{1} \quad 4 \quad 4 \quad -16 \quad 24 \\ \hline \end{array}$$

$$\begin{array}{r} -3 \mid 1 \quad 1 \quad -4 \quad 6 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} \phantom{-3 \mid} \phantom{1} \quad -3 \quad 6 \quad -6 \\ \hline \end{array}$$

$$\begin{array}{r} \phantom{-3 \mid} \phantom{1} \quad 1 \quad -2 \quad 2 \quad 0 \\ \hline \end{array}$$

$$(x-4)(x+3)(x^2-2x+2) = 0$$

$$\boxed{x=4} \quad \boxed{x=-3} \quad x = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = \boxed{1 \pm i}$$

$$10. \quad 12x^3 + x^2 - 21x + 5 = 0$$

$$\begin{array}{r|l} \text{Max \# +} & + 2 \quad | \quad 0 \\ \hline & - 1 \quad | \quad 1 \end{array}$$

$$f(-x) = -12x^3 + x^2 + 21x + 5$$

$$\begin{array}{r|l} \text{Max \# -} & i \quad 0 \quad | \quad 2 \\ \hline & \end{array}$$

$$\begin{array}{r} \frac{1}{4} \mid 12 \quad 1 \quad -21 \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} \phantom{\frac{1}{4} \mid} \phantom{12} \quad 3 \quad 1 \quad -5 \\ \hline \end{array}$$

$$\begin{array}{r} \phantom{\frac{1}{4} \mid} \phantom{12} \quad 12 \quad 4 \quad -20 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} \phantom{\frac{1}{4} \mid} \phantom{12} \quad 4 \\ \hline \end{array}$$

$$(4x-1)(3x^2+x-5) = 0$$

$$\boxed{x = \frac{1}{4}} \quad x = \frac{-1 \pm \sqrt{1+60}}{6} = \boxed{\frac{-1 \pm \sqrt{61}}{6}}$$

\* correction

$$11. \quad 10x^3 - 3x^2 + 3x - 2 = 0$$

$$\begin{array}{r|l} \text{Max \# +} & + 3 \quad | \quad 1 \\ \hline & - 0 \quad | \quad 0 \end{array}$$

$$f(-x) = -10x^3 - 3x^2 - 3x - 2$$

$$\begin{array}{r|l} \text{Max \# -} & i \quad 0 \quad | \quad 2 \\ \hline & \end{array}$$

$$\begin{array}{r} \frac{1}{2} \mid 10 \quad -3 \quad 3 \quad -2 \\ \hline \end{array}$$

$$\begin{array}{r} \phantom{\frac{1}{2} \mid} \phantom{10} \quad 5 \quad 1 \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} \phantom{\frac{1}{2} \mid} \phantom{10} \quad 10 \quad 2 \quad 4 \quad 0 \\ \hline \end{array}$$

$$(2x-1)(5x^2+x+2) = 0$$

$$\boxed{x = \frac{1}{2}}$$

$$x = \frac{-1 \pm \sqrt{1-40}}{10} = \frac{-1 \pm i\sqrt{39}}{10}$$