Honors Algebra II Name_____

Unit 4B: Polynomial Functions **Fundamental Theorem of Algebra Practice 3**

Verify that the zero given is a factor of the polynomial. Then find the remaining zeros.

1. $x=5i, f(x)=2x^3+3x^2+50x+75$
2. $x=2i, f(x)=2x^4-x^3+7x^2-4x-4$
3. $x=5i, f(x)=x^3-x^2+25x-25$
4. $x=5+2i, f(x)=x^3-7x^2-x+87$
5. $x=4+2i, f(x)=x^3-5x^2-4x+60$
6. $x=3-2i, f(x)=x^3-9x^2+31x-39$
7. $x=1-3i, f(x)=x^3+6x+20$
8. $x=1+i\sqrt{3}, f(x)=x^4-2x^3+10x^2-12x+24$

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include those given.

9. 2 (w/multiplicity of two), i
10. 4, $2-3i$
11. $1+3i$, $3-i$
12. -2, $1+\sqrt{5}$
13. 2, $1-\sqrt{3}$
14. -2, $\frac{1}{3}$, $2+3i$
15. $4-i$, $1+\sqrt{3}$
16. $\frac{2}{3}$, $1-i$, $i\sqrt{2}$

Use Descartes' Rule of Signs to determine the possible numbers of positive and negative real zeros of the function. Then determine the possible combinations of positive, negative and imaginary solutions.

17. $f(x)=x^3+x^2-x+1$
18. $f(x)=x^3+x^2+x+1$
19. $f(x)=2x^3+x-3$
20. $f(x)=5x^4+x^2-3x-2$
21. $f(x)=x^5-x^3+x^2-3$
22. $f(x)=2x^6-x^4+3x+13$
23. $f(x)=x^5-x^4+5x^3-3x^2+4x-6$
24. $f(x)=x^5+2x^4-6x^2-3x-9$

1. $x = 5i$, $f(x) = 2x^3 + 3x^2 + 50x + 75$

$5i$	2	3	50	75
		$10i$	$-50+15i$	-75
$-5i$	2	$3+10i$	$15i$	0
		$-10i$	$-15i$	
	2	3	0	

$(x-5i)(x+5i)(2x+3)$; $\boxed{x = \pm 5i, -\frac{3}{2}}$

2. $x = 2i$; $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

$2i$	2	-1	7	-4	-4
		$4i$	$-8-2i$	$4-2i$	4
$-2i$	2	$-1+4i$	$-1-2i$	$-2i$	0
		$-4i$	$2i$	$2i$	
	2	-1	-1	0	

$2x^2 - x - 1$

$(x-2i)(x+2i)(2x+1)(x-1)$; $\boxed{x = 2i, -2i, 1, -\frac{1}{2}}$

$\frac{+1 \pm \sqrt{9}}{4}$

3. $x = 5i$; $f(x) = x^3 - x^2 + 25x - 25$

$5i$	1	-1	25	-25
		$5i$	$-25-5i$	25
$-5i$	1	$-1+5i$	$-5i$	0
		$-5i$	$5i$	
	1	-1	0	

$(x-5i)(x+5i)(x-1) = 0$; $\boxed{x = \pm 5i, 1}$

4. $5+2i$ | 1 -7 -1 87

$5+2i$ $\quad -14+6i \quad -87$

$5-2i$ | $1 \quad -2+2i \quad -15+6i \quad 0$

$5-2i \quad 15-6i$

$1 \quad 3 \quad 0$

$(x-5-2i)(x-5+2i)(x+3)$; $\boxed{x=5\pm 2i, -3}$

5. $4+2i$ | 1 -5 -4 60

$4+2i \quad -8+6i \quad -60$

$4-2i$ | $1 \quad -1+2i \quad -12+6i \quad 0$

$4-2i \quad 12-6i$

$1 \quad 3 \quad 0$

$(x-4-2i)(x-4+2i)(x+3)=0$; $\boxed{x=4\pm 2i, -3}$

6. $3-2i$ | 1 -9 31 -39

$3-2i \quad -22+6i \quad 39$

$3+2i$ | $1 \quad -6-2i \quad 9+6i \quad 0$

$3+2i \quad -9-6i$

$1 \quad -3 \quad 0$

$(x-3+2i)(x-3-2i)(x-3)$; $\boxed{x=3\pm 2i, 3}$

7. $1-3i$ | 1 0 6 20

$1-3i \quad -8-6i \quad -20$

$1+3i$ | $1 \quad 1-3i \quad -2-6i \quad 0$

$1+3i \quad 2+6i$

$1 \quad 2 \quad 0$

$(1-3i)(1-3i)$

$1-6i+9i^2$

$-1(1+i\sqrt{3})(1-i\sqrt{3})$

$-1(1+3)$

$(x-1+3i)(x-1-3i)(x+2)$; $\boxed{x=1\pm 3i; -2}$

$(-6+6i\sqrt{3})(1+i\sqrt{3})$

8. $1+i\sqrt{3}$ | 1 -2 10 -12 24

$1+i\sqrt{3} \quad -4 \quad 6+6i\sqrt{3} \quad -24$

$-6-6i\sqrt{3}+6i\sqrt{3}$

-18

$1-i\sqrt{3}$ | $1 \quad -1+i\sqrt{3} \quad 6 \quad -6+6i\sqrt{3} \quad 0$

$1-i\sqrt{3} \quad 0 \quad 6-6i\sqrt{3}$

$1 \quad 0 \quad 6 \quad 0$

$\boxed{x=1\pm i\sqrt{3}, \pm i\sqrt{6}}$

9. $(x-2)^2(x+i)(x-i)$

$(x^2-4x+4)(x^2+1)$

$\boxed{P(x) = x^4 - 4x^3 + 5x^2 - 4x + 4}$

10. $(x-4)(x-2+3i)(x-2-3i)$

	x	-2	$+3i$
x	x^2	$-2x$	~~$3xi$~~
-2	$-2x$	4	~~$-6i$~~
$-3i$	~~$-3xi$~~	~~$6i$~~	$-9i^2$

$= x^2 - 4x + 13$

$(x-4)(x^2-4x+13)$

$\boxed{P(x) = x^3 - 8x^2 + 29x - 52}$

11. $(x-1-3i)(x-1+3i)(x-3+i)(x-3-i)$

	x	-1	$-3i$
x	x^2	$-x$	$-3xi$
-1	$-x$	1	$3i$
$3i$	$3xi$	$-3i$	$-9i^2$

	x	-3	i
x	x^2	$-3x$	xi
-3	$-3x$	9	$-3i$
$-i$	$-xi$	$3i$	$-i^2$

$(x^2-2x+10)(x^2-6x+10)$

	x^2	$-2x$	10
x^2	x^4	$-2x^3$	$10x^2$
$-6x$	$-6x^3$	$12x^2$	$-60x$
10	$10x^2$	$-20x$	100

$\boxed{P(x) = x^4 - 8x^3 + 32x^2 - 80x + 100}$

12. $(x+2)(x-1-\sqrt{5})(x-1+\sqrt{5})$

	x	-1	$-\sqrt{5}$
x	x^2	$-x$	$-x\sqrt{5}$
-1	$-x$	1	$\sqrt{5}$
$\sqrt{5}$	$x\sqrt{5}$	$-\sqrt{5}$	-5

$= x^2-2x-4$

$(x+2)(x^2-2x-4)$

$\boxed{P(x) = x^3-8x-8}$

-2	1	0	-8	-8
	-2	4	8 ✓
1	-2	-4	$\boxed{0}$

13. $(x-2)(x-1+\sqrt{3})(x-1-\sqrt{3})$

	x	-1	$\sqrt{3}$
x	x^2	$-x$	$x\sqrt{3}$
-1	$-x$	1	$-\sqrt{3}$
$-\sqrt{3}$	$-x\sqrt{3}$	$\sqrt{3}$	-3

$= x^2-2x-2$

$(x-2)(x^2-2x-2)$

$\boxed{P(x) = x^3-4x^2+2x+4}$

14. $(x+2)(3x-1)(x-2-3i)(x-2+3i)$

	x	-2	$-3i$
x	x^2	$-2x$	$-3xi$
-2	$-2x$	4	$6i$
$3i$	$3xi$	$-6i$	$-9i^2$

$= x^2-4x+13$

$(3x^2+5x-2)(x^2-4x+13)$

	$3x^2$	$5x$	-2
x^2	$3x^4$	$5x^3$	$-2x^2$
$-4x$	$-12x^3$	$-20x^2$	$8x$
13	$39x^2$	$65x$	-26

$\boxed{P(x) = 3x^4-7x^3+17x^2+73x-26}$

15. $(x-4+i)(x-4-i)(x-1-\sqrt{3})(x-1+\sqrt{3})$

work on #13

	x	-4	$+i$
x	x^2	$-4x$	xi
-4	$-4x$	16	$-4i$
$-i$	$-xi$	$4i$	$-i^2$

$(x^2-8x+17)(x^2-2x-2)$

	x^2	$-8x$	17
x^2	x^4	$-8x^3$	$17x^2$
$-2x$	$-2x^3$	$16x^2$	$-34x$
-2	$-2x^2$	$16x$	-34

$$\boxed{P(x)=x^4-10x^3+31x^2-18x-34}$$

16. $(3x-2)(x-1+i)(x-1-i)(x-i\sqrt{2})(x+i\sqrt{2})$

	x	-1	$+i$
x	x^2	$-x$	xi
-1	$-x$	1	$-i$
$-i$	$-xi$	i	$-i^2$

$x=\pm i\sqrt{2}$

$x^2=i^2(2)$

$x^2=-2$

$x^2+2=0$

$(3x-2)(x^2-2x+2)(x^2+2)$

$(3x^3-2x^2+6x-4)(x^2-2x+2)$

	$3x^3$	$-2x^2$	$6x$	-4
x^2	$3x^5$	$-2x^4$	$6x^3$	$-4x^2$
$-2x$	$-6x^4$	$4x^3$	$-12x^2$	$8x$
2	$6x^3$	$-4x^2$	$12x$	-8

$$\boxed{P(x)=3x^5-8x^4+16x^3-20x^2+20x-8}$$

17. $f(x) = x^3 + x^2 - x + 1$ (x, 1, 2)

2 or 0 pos. roots

$f(-x) = -x^3 + x^2 + x + 1$ (1, x, x)

1 neg. root

+	2	0
−	1	1
i	0	2

18. $f(x) = x^3 + x^2 + x + 1$ (x, x, x)

0 pos. roots

$f(-x) = -x^3 + x^2 - x + 1$ (1, 2, 3)

3 or 1 neg. roots

+	0	0
−	3	1
i	0	2

19. $f(x) = 2x^3 + x - 3$ (x, 1)

1 pos. root

$f(-x) = -2x^3 - x - 3$ (x, x)

0 neg. roots

+	1
−	0
i	2

20. $f(x) = 5x^4 + x^2 - 3x - 2$ (x, 1, x)

1 pos. root

$f(-x) = 5x^4 + x^2 + 3x - 2$ (x, x, 1)

1 neg. root

+	1
−	1
i	2

21. $f(x) = x^5 - x^3 + x^2 - 3$ $\quad f(-x) = -x^5 + x^3 + x^2 - 3$

(1 2 3) (1 x 2)

3 or 1 pos. zeros $\quad$ 2 or 0 neg. zeros

+	3	3	1	1
−	2	0	2	0
i	0	2	2	4

22. $f(x) = 2x^6 - x^4 + 3x + 13$ $\quad f(-x) = 2x^6 - x^4 - 3x + 13$

(1 2 x) (1 x 2)

2 or 0 pos. zeros $\quad$ 2 or 0 neg. zeros

+	2	2	0	0
−	2	0	2	0
i	2	4	4	6

23. $f(x) = x^5 - x^4 + 5x^3 - 3x^2 + 4x - 6$ $\quad f(-x) = -x^5 - x^4 - 5x^3 - 3x^2 - 4x - 6$

(1 2 3 4 5) (x x x x x)

5, 3, or 1 pos. zero $\quad$ 0 neg. zeros

+	5	3	1
−	0	0	0
i	0	2	4

24. $f(x) = x^5 + 2x^4 - 6x^2 - 3x - 9$ $\quad f(-x) = -x^5 + 2x^4 - 6x^2 + 3x - 9$

(x 1 x x) (1 2 3 4)

1 pos zero $\quad$ 4, 2, or 0 neg. zeros

+	1	1	1
−	4	2	0
i	0	2	4