Honors Algebra II **Name**____________________________________

WS – 9: Exponential and Logarithm Application Problems

Set up each problem and show your work.

1. The exponential growth model $P = 5344e^{0.012744t}$ approximates the world population (in millions) from 1990. According to this model, when will the world population reach 6.8 billion?

2. Diane invests \$8500 at 8.5% annual interest, compounded quarterly. How long will it take for the account to double in value?

3. In a research experiment, a population of fruit flies is increasing according a normal exponential model. After 2 days there are 100 flies. After 4 days there are 300 flies. How many flies will there be after 5 days?

4. An isotope had a half life of 80 days. How many days will it take of the 7 mg sample of this isotope to decay to 1 mg? Round to the nearest whole number.

5. The yield in millions of cubic feet of trees per acre is given by $y = 6.7e^{\frac{-48.1}{t}}$. Find the yield after a) 15 years and b) 50 years.

6. A satellite has a radioisotope power supply. The power output in watts is given by the equation $P = 50e^{\frac{-t}{250}}$, where t is the time in days since the power supply was placed in service.
   a) How much power will be available at the end of one year?
   b) The equipment aboard the satellite requires 10 watts of power to operate properly. What is the operational life of the satellite?

7. The United States public debt, in billions of dollars, has been estimated with the model $y = 0.051517(1.1306727)^x$. The exponent represents the number of years since 1900.
   a) According to the model, when did the debt pass \$1 trillion (\$1000 billion).
   b) According to the model, what is the annual growth rate of the debt?

8. Carbon 14 has a half life of 5730 years. Use the half-life model to answer the following:
   a) What is the value of k in this problem?
   b) How old is an animal bone that has lost 30% of its carbon 14?
   c) A mummy discovered in the pyramid Khufu in Egypt has lost 46% of its carbon 14. Determine its age.

9. DDT is an insecticide that has been used by farmers. It decays slowly and is sometimes absorbed by plants that animals and humans eat. DDT absorbed in the mud at the bottom of a lake is degraded into harmless products by bacterial action. Experimental data shows that 10% of the initial amount is eliminated in 5 years.
   a) Find the k value in the decay formula.
   b) How much of the original amount of DDT is left after 10 years?
   c) The US Environmental Protection Agency banned almost all use of DDT in the US in 1972. If none has been used near the lake since then, in what year will the concentrations of DDT fall below 25%?

10. If \$1000 is deposited at an annual rate of 8.25% compounded continuously, how long will it take for the account to double in value?

11. A typical nuclear power plant produces about 10 lb of Krypton-85 per year. The half-life of Krypton-85 is 11 years. How long must the Krypton be contained so only 0.1 lb will remain?

1. $P = 5344e^{.012744t}$

6.8 billion = 6800 million

$6800 = 5344e^{.012744t}$

$1.272 = e^{.012744t}$

$\ln 1.272 = .012744t$

$t = \frac{\ln 1.272}{.012744} = 18.91$ years from 1990, during 2008

2. $A = P(1 + \frac{r}{n})^{nt}$

$2 = (1 + \frac{.085}{4})^{4t}$

$2 = 1.02125^{4t}$

$4t = \log_{1.02125} 2$

$t = \frac{\log_{1.02125} 2}{4} = 8.24$ years

3. $P = P_0 e^{kt}$

$300 = 100e^{2k}$

$3 = e^{2k}$

$\ln 3 = 2k$

$k = \frac{\ln 3}{2} = .549$

$P = 100e^{.549(3)}$

$P = 519.62 \approx 520$ fruit flies

4. $1 = 7\left(\frac{1}{2}\right)^{t/80}$

$\frac{1}{7} = \left(\frac{1}{2}\right)^{t/80}$

$\frac{t}{80} = \log_{\frac{1}{2}}\left(\frac{1}{7}\right)$

$t = 80 \log_{\frac{1}{2}}\left(\frac{1}{7}\right) = 224.59 \approx 225$ days

5. $y = 6.7e^{-48.1/t}$

a) $y = 6.7e^{-48.1/15} = 0.27$ millions of ft^3 of trees

b) $y = 6.7e^{-48.1/50} = 2.56$ millions of ft^3 of trees

6. $P = 50e^{-t/250}$

a) $P = 50e^{-365/250} = 11.61$ watts

b) $10 = 50e^{-t/250}$

$.2 = e^{-t/250}$

$\ln .2 = -t/250$

$250 \ln .2 = -t$

$t = -250 \ln .2 = 402.36 \approx 402$ days

7. $y = .051517(1.1306727)^x$

a. $1000 = .051517(1.1306727)^x$

$19411.07 = 1.1306727^x$

$x = \log_{1.1306727}(19411.07) = 80.40$ years, so in the year 1980

b. $1 + r = 1.1306727$

$r = .1306727$

The annual growth rate is 13.06727%

8. $A = A_0\left(\frac{1}{2}\right)^{t/h}$

b) $.7 = \frac{1}{2}^{t/5730}$

$\frac{t}{5730} = \log_{\frac{1}{2}} .7$

$t = 5730 \log_{\frac{1}{2}}(.7) = 2948.50 \approx 2950$ years old

c) $.54 = \frac{1}{2}^{t/5730}$

$\frac{t}{5730} = \log_{\frac{1}{2}} .54$

$t = 5730 \log_{\frac{1}{2}}(.54) = 5093.79 \approx 5100$ years old

8a. $\frac{1}{2} = e^{5730k}$

$\ln \frac{1}{2} = 5730k$

$k = \frac{\ln \frac{1}{2}}{5730}$

$k = -.000121$

b. $.7 = e^{-.000121t}$

$\ln .7 = -.000121t$

$t = \frac{\ln .7}{-.000121} = 2948.50 \approx 2950$ years

c. $.54 = e^{-.000121t}$

$\ln .54 = -.000121t$

$t = \frac{\ln .54}{-.000121} = 5094 \approx 5100$ years

9. $P = P_0 e^{kt}$

$.9 = e^{5k}$

$\ln .9 = 5k$

$k = \frac{\ln .9}{5} = -.0211$

b. $P = e^{-.0211(10)}$

$P = .81 = 81\%$ of the original amount

c. $.25 = e^{-.0211t}$

$\ln .25 = -.0211t$

$t = 65.79$ years

$1972 + 65.79 = 2037.78$ during the year 2037

10. $2 = e^{.0825t}$

$\ln 2 = .0825t$

$t = \frac{\ln 2}{.0825} = 8.4$ years

11. $0.1 = 10\left(\frac{1}{2}\right)^{t/11}$

$.01 = \left(\frac{1}{2}\right)^{t/11}$

$\log_{\frac{1}{2}}(0.01) = \frac{t}{11}$

$t = 11 \log_{\frac{1}{2}}(0.01)$

$t = 73.08$ years