

Real World Application (from Math 3 book and note-taking guide)

1. From 1990 to 2006, the population  $P$  (in millions) of a large city can be modeled by  $P = 1.1(1.05)^t$  where  $t$  is the number of years since 1990.

- A. What is the population in 1990? 1.1 million people  
 B. What is the percent of increase each year? 5%  
 C. What is the expected population in 2002?  $P = 1.1(1.05)^{12} = 1.98$  million people

2. The number of applications to Georgia Tech was 35000 in 2015 and growing at a rate of 4.6% every 2 years. How many applications does Georgia Tech expect to receive in 2022?

$$f(7) = 35000(1.046)^{7/2} = 40966.51 \approx 40967 \text{ applications}$$

3. You buy a car for \$12,500. You had to take out a loan for the full amount. The loan you got is a 72 month loan. The interest rate on the loan is 2.85% compounded monthly.

- A. What is the amount you end up paying for the car once you are done paying the loan off?  $y = 12500(1 + \frac{0.0285}{12})^{72} = \$14828.13$   
 B. How much money could you have saved if you paid off the loan in half the time?

$$y = 12500(1 + \frac{0.0285}{12})^{36} = \$13614.39 \quad \text{You could save } \$1213.74$$

4. A population of 500 elk is released in a wildlife preserve. Every 3 years, the population grows by 16.4%.

- A. Write an exponential equation that represents the number of elk  $x$  years after the release.

$$f(x) = 500(1.164)^{x/3}$$

- B. After 5 years, how many elk are there?

$$f(5) = 500(1.164)^{5/3} = 644.01 \approx 644 \text{ elk}$$

5. Suppose the population of a nation is growing by 9% per decade. If the population was 30,000,000 in 1975, what will the population be in 2019, to the nearest million?

$$f(44) = 30,000,000(1.09)^{44/10} = 43832661.91 \approx 44,000,000 \text{ people}$$

6. According to legend, in 1626 Manhattan Island was purchased for trinkets worth about \$24. If the \$24 had been invested at a rate of 6% interest per year, what would be its value in 2006? Compare this with a total of \$802.4 billion in assessed values for Manhattan in 2006.

$$f(380) = 24(1.06)^{380} = \$99.18 \text{ billion} \quad \text{About } \frac{1}{10} \text{ as much as the actual value}$$

7. If the price of theater tickets increases by 2% 3 times per year, how much will a \$100 ticket cost 5 years from now?

$$f(5) = 100(1.02)^{3(5)} = \$134.59$$

8. You bought a new car for \$28,000. The car depreciates (loses value) at a rate of 15% per year.

- A. What is the exponential decay model giving the car's value  $V$  after  $t$  years?  $V(t) = 28000(.85)^t$   
 B. What is the value of the car after 3.5 years?  $V(3.5) = 28000(.85)^{3.5} = \$15853.47$   
 C. After about how many years will the car be worth \$10? About 48.5 years

5	10	15	20	25	45	48	49
12423.75	5512.48	2445.92	1085.27	481.54	18.66	11.46	9.74

9. You charged \$200 to your credit card. Your credit card charges you 8.9% per month. Assume the card doesn't charge you late fees.

A. If you don't make a payment, how much will you owe after 5 months? How much more than your original charge did you pay in interest?

$y = 200(1.089)^5 = \$306.32$       \$106.32 in interest

B. If you don't make a payment, how much will you owe after 2 years? How much more than your original charge did you pay in interest?

$y = 200(1.089)^{24} = \$1547.74$       \$1347.74 in interest

Use the compound interest formulas for problems 10 and 11.

$A = P(1 + \frac{r}{n})^{nt}$ ;  $A = Pe^{rt}$

10. You are saving \$3000 at 5% compounded monthly.

A. How much do you have after 5 years?  $A = 3000(1 + \frac{0.05}{12})^{12 \cdot 5} = \$3850.08$

B. How much do you have after 20 years?

$A = 3000(1 + \frac{0.05}{12})^{12 \cdot 20} = \$8137.92$

C. If the interest were compounded continuously, how much would you have after 20 years?

$A = 3000e^{(0.05)(20)} = \$8154.85$       \$16.93 more

11. You deposit \$1000 in an account that earns 3% annual interest.

A-D. Find the balance for the given number of years if the interest is compounded with the given frequency. (show your set up)

	A.	B.	C.	D.
# of years:	3	10	21	4
Compounded:	quarterly	Annually	monthly	continuously

\$3091.02      \$1343.92      \$1876.14      \$1127.50

E. How much more would you earn on your \$1000 compounding continuously for 20 years at a rate of 3% than you would compounding quarterly?

$A = 1000e^{0.03 \cdot 20} = \$1822.12$        $A = 1000(1 + \frac{0.03}{4})^{4 \cdot 20} = \$1818.04$       \$4.08 more

12. The half-life of strontium-90 (Sr-90) is approximately 28 years. The amount A (in milligrams) of Sr-90 that remains in a sample after t years is given by

$A = 120(\frac{1}{2})^{t/28}$

A. What is the initial amount? 120 mg

B. In about how many years will 30 mg of the Sr-90 remain?

$30 = 120(\frac{1}{2})^{t/28}$   
 $\frac{1}{4} = \frac{1}{2}^{t/28}$   
 $(\frac{1}{2})^2 = (\frac{1}{2})^{t/28}$   
 $2 = \frac{t}{28}$   
 $t = 56$  years

C. How much Sr-90 will be present after 7 years?

$A = 120(\frac{1}{2})^{7/28} = 100.91$  mg

D. After how many years will you have 1/2 of your initial amount?

28 years (coefficient)

13. The table below represents the number of people using beepers. Consider 2000 as yr 0.

A. Write an equation to represent the number of beepers being used according to the year 2000.

$y = 100,000(.7)^x$

B. Based on your equation, how many beepers will be used in 2020?

$y = 100,000(.7)^{20} = 79.79 \approx 80$  beepers

C. Based on your equation, when did the beeper usage decrease faster: between 2000 and 2005 or between 2005 and 2010?

Year	# of beepers
2000	100,000
2001	70,000
2002	49,000
2003	34,300

ROC  $\frac{16807 - 100000}{2005 - 2000} = -16638.6$  beepers/yr      ROC  $\frac{2824.75 - 16807}{2010 - 2005} = -2796.45$  beepers/yr

CHALLENGE: The cost of goods and services in an urban area increased by 1.5% last month. If this rate continues, what will be the annual rate of increase?

$(1 + 0.015)^{12} = 1.20$

20% annually