For each function:

a) List the possible rational roots of f

b) Use synthetic division and the possible roots to find a factor of the polynomial

c) Factor the polynomial completely

1. $f(x) = x^3 + x^2 - 4x - 4$

2. $f(x) = -3x^3 + 20x^2 - 36x + 16$

3. $f(x) = -4x^3 + 15x^2 - 8x - 3$

4. $f(x) = 4x^4 - 17x^2 + 4$

5. $f(x) = 6x^3 - x^2 - 13x + 8$

6. $f(x) = x^4 - x^3 - 2x - 4$

7. $f(x) = 2x^4 - 7x^3 - 2x^2 - 7x - 4$

8. $f(x) = 2x^4 - 11x^3 - 6x^2 + 64x + 32$

WS - Rational Root Theorem and Factoring

1. $f(x) = x^3 + x^2 - 4x - 4$

a. $\dfrac{p = 4}{1, 2, 4}$

$\dfrac{q = 1}{1}$

$\boxed{\pm 1, \pm 2, \pm 4}$

b.

$$-1 \begin{array}{|rrrr} 1 & 1 & -4 & -4 \\ & -1 & 0 & 4 \\ \hline 1 & 0 & -4 & \boxed{0} \end{array}$$

$\Rightarrow (x+1)(x^2 - 4)$

c. $f(x) = (x+1)(x+2)(x-2)$

2. $f(x) = -3x^3 + 20x^2 - 36x + 16 = -1(3x^3 - 20x^2 + 36x - 16)$

a. $\dfrac{p = 16}{1, 2, 4, 8, 16}$

$\dfrac{q = 3}{1, 3}$

$\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}, \pm 8, \pm \frac{8}{3}, \pm 16, \pm \frac{16}{3}$

b.

$$1 \begin{array}{|rrrr} 3 & -20 & 36 & -16 \\ & 3 & -17 & \\ \hline 3 & -17 & 19 & \end{array}$$

$$2 \begin{array}{|rrrr} 3 & -20 & 36 & -16 \\ & 6 & -28 & 16 \\ \hline 3 & -14 & 8 & \boxed{0} \end{array}$$

$\Rightarrow (x-2)(3x^2 - 14x + 8)$

c. $f(x) = -1(x-2)(3x^2 - 14x + 8)$

$\quad = -1(x-2)(3x^2 - 2x - 12x + 8)$

$\quad = -1(x-2)(x-4)(3x-2)$

$$\begin{array}{c} 24 \\ -12 \diagdown -2 \\ -14 \end{array}$$

3. $f(x) = -4x^3 + 15x^2 - 8x - 3 = -1(4x^3 - 15x^2 + 8x + 3)$

a. $p = 3$
   $\overline{1, 3}$

   $q = 4$
   $\overline{1, 2, 4}$

   $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

b.
$$\begin{array}{c|cccc} 1\rfloor & 4 & -15 & 8 & 3 \\ & & 4 & -11 & -3 \\ \hline & 4 & -11 & -3 & \boxed{0} \end{array} \Rightarrow (x-1)(4x^2 - 11x - 3)$$

c. $f(x) = -1(x-1)(4x^2 - 11x - 3)$
   $= -1(x-1)(4x - 12x + 1x - 3)$
   $= -1(x-1)(4x+1)(x-3)$

-12
$-12 \diagdown -1$
$\diagdown -11$

4. $f(x) = 4x^4 - 17x^2 + 4$

a. $p = 4$
   $\overline{1, 2, 4}$ $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \cancel{\pm \frac{2}{2}}, \cancel{\pm \frac{2}{4}}, \pm 4, \cancel{\pm \frac{4}{2}}, \cancel{\pm \frac{4}{4}}$

   $q = 4$
   $\overline{1, 2, 4}$ $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 4$

b.
$$\begin{array}{c|ccccc} 2\rfloor & 4 & 0 & -17 & 0 & 4 \\ & & 8 & 16 & -2 & -4 \\ \hline -2\rfloor & 4 & 8 & -1 & -2 & \boxed{0} \\ & & -8 & 0 & 2 & \\ \hline & 4 & 0 & -1 & \boxed{0} & \end{array} \Rightarrow (x-2)(x+2)(4x^2 - 1)$$

c. $f(x) = (x-2)(x+2)(2x-1)(2x+1)$

5. $f(x) = 6x^3 - x^2 - 13x + 8$

a. $p = 8$

   $1, 2, 4, 8$

   $q = 6$

   $1, 2, 3, 6$

   $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{2}, \pm \frac{2}{3}, \pm \frac{2}{6}$

   $\pm 4, \pm \frac{4}{2}, \pm \frac{4}{3}, \pm \frac{4}{6}, \pm 8, \pm \frac{8}{2}, \pm \frac{8}{3}, \pm \frac{8}{6}$

   $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}, \pm 8, \pm \frac{8}{3}$

b.
$$\begin{array}{c|cccc} 1] & 6 & -1 & -13 & 8 \\ & & 6 & 5 & -8 \\ \hline & 6 & 5 & -8 & \boxed{0} \end{array}$$
$\Rightarrow (x-1)(6x^2 + 5x - 8)$

c. $f(x) = (x-1)(6x^2 + 5x - 8)$

   not factorable

   $\frac{-48}{5}$ (crossed out)

   $(5)^2 - 4(6)(-8)$

   $= 217$

6. $f(x) = x^4 - x^3 - 2x - 4$

a. $p = 4$

   $1, 2, 4$

   $q = 1$

   1

   $\pm 1, \pm 2, \pm 4$

b.
$$\begin{array}{c|ccccc} 1] & 1 & -1 & 0 & -2 & -4 \\ & & 1 & 0 & 0 & -2 \\ \hline & 1 & 0 & 0 & -2 & \boxed{-6} \end{array}$$

$$\begin{array}{c|ccccc} -1] & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & \boxed{0} \end{array}$$
$\Rightarrow (x+1)(x^3 - 2x^2 + 2x - 4)$

c. $f(x) = (x+1)(x^3 - 2x^2 + 2x - 4)$

   $= (x+1)(x^2 + 2)(x - 2)$

7. $f(x) = 2x^4 - 7x^3 - 2x^2 - 7x - 4$

$p = 4$

$1, 2, 4$

$\pm 1, \pm \frac{1}{2}, \pm 2, \pm\frac{2}{2}, \pm 4, \pm\frac{4}{2}$

$q = 2$

$1, 2$

$\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4$

b.

$\underline{1)}$
2	-7	-2	-7	-4
	2	-5	-7	
2	-5	-7		

$\underline{-2)}$
2	-7	-2	-7	-4
	-4	22	-40	
2	-11	20		

$\underline{-1)}$
2	-7	-2	-7	-4
	-2	9	-7	
2	-9	7		

$\underline{2)}$
2	-7	-2	-7	-4
	4	-6	16	
2	-3	-8	9	

$\underline{\frac{1}{2})}$
2	-7	-2	-7	-4
	1	-3		
2	-6	-5		

$\underline{-\frac{1}{2})}$
2	-7	-2	-7	-4
	-1	4	-1	4
2	-8	2	-8	$\boxed{0}$

$\div 2$

$(2x+1)(x^3 - 4x^2 + x - 4)$

c. $f(x) = (2x+1)(x^3 - 4x^2 + x - 4)$
$= (2x+1)(x^2 + 1)(x - 4)$

8. $f(x) = 2x^4 - 11x^3 - 6x^2 + 64x + 32$

a. $\underline{p = 32}$
$\quad 1, 2, 4, 8, 16, 32$

$\quad \underline{q = 2}$
$\quad 1, 2$

$\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

b.
$1 \rfloor \begin{array}{ccccc} 2 & -11 & -6 & 64 & 32 \\ & 2 & -9 & -15 & \\ \hline 2 & -9 & -15 & & \end{array}$

$-1 \rfloor \begin{array}{ccccc} 2 & -11 & -6 & 64 & 32 \\ & -2 & 13 & -7 & \\ \hline 2 & -13 & 7 & & \end{array}$

$2 \rfloor \begin{array}{ccccc} 2 & -11 & -6 & 64 & 32 \\ & 4 & -14 & -40 & 48 \\ \hline 2 & -7 & -20 & 24 & \end{array}$

$-2 \rfloor \begin{array}{ccccc} 2 & -11 & -6 & 64 & 32 \\ & -4 & 30 & -48 & -32 \\ \hline 2 & -15 & 24 & 16 & \boxed{0} \end{array}$

$-2 \rfloor \begin{array}{cccc} 2 & -15 & 24 & 16 \\ & -4 & 38 & \\ \hline 2 & -19 & & \end{array}$

$\frac{1}{2} \rfloor \begin{array}{cccc} 2 & -15 & 24 & 16 \\ & 1 & -7 & \\ \hline 2 & -14 & 17 & \end{array}$

$-\frac{1}{2} \rfloor \begin{array}{cccc} 2 & -15 & 24 & 16 \\ & -1 & 8 & -16 \\ \hline 2 & -16 & 32 & \boxed{0} \end{array}$

$\quad\quad\quad\quad\quad 2$

$(x+2)(2x+1)(x^2 - 8x + 16)$

c. $f(x) = (x+2)(2x+1)(x-4)^2$