p.163, #5-10, 22-25

5. $2, 6, 18, 54, 162, \ldots$

$a_n = 2(3)^{n-1}$

$a_{10} = 2(3)^{10-1} = \boxed{39,366}$

6. $5000, 500, 50, 5, 0.5, \ldots$

$a_n = 5000(0.1)^{n-1}$

$a_{10} = 5000(0.1)^{10-1} = \boxed{0.000005}$ or $\frac{5}{1000000} = \boxed{\frac{1}{200000}}$

7. $-0.125, 0.25, -0.5, 1, -2$

$r = \frac{0.25}{-0.125} = -2$

$a_n = -0.125(-2)^{n-1}$

$a_{10} = -0.125(-2)^{10-1} = \boxed{64}$

8. $a_4 = -12,\ a_5 = -4$

$r = \frac{-4}{-12} = \frac{1}{3}$

$a_6 = -4\left(\frac{1}{3}\right) = \boxed{\frac{-4}{3}}$

9. $a_2 = 4,\ a_5 = 108$

$a_n = a_2(r)^{n-2}$

$a_5 = a_2(r)^{5-2}$

$108 = 4(r)^3$

$27 = r^3$

$r = 3$

$a_6 = 108 \cdot 3$

$\boxed{a_6 = 324}$

10. $a_3 = 3,\ a_5 = 12$

$a_5 = a_3(r)^{5-3}$

$12 = 3r^2$

$4 = r^2$

$r = \pm 2$

$a_5 = 12,\ r = 2$

$a_6 = 12 \cdot 2 = \boxed{24}$

$a_5 = 12,\ r = -2$

$a_6 = 12 \cdot -2 = \boxed{-24}$

pg. 163-164, #19-28, 32-42, 44-46, 49.

19. $-36, -49, -64, -81, \ldots$

neither

20. $-2, -6, -18, -54, \ldots$

geometric

$r = 3$

21. $2, 7, 12, 17, \ldots$

arithmetic

$d = 5$

22. $\frac{1}{2}, \frac{1}{10}, \frac{1}{50}, \frac{1}{250}, \frac{1}{1250}$

$a_n = \frac{1}{2}\left(\frac{1}{5}\right)^{n-1}$

$a_9 = \frac{1}{2}\left(\frac{1}{5}\right)^{9-1} = \boxed{\frac{1}{781250}}$

23. $3, -6, 12, -24, 48$

$a_n = 3(-2)^{n-1}$

$a_9 = 3(-2)^{9-1}$

$= \boxed{768}$

24. $3200, 1600, 800, 400, 200, \ldots$

$a_n = 3200\left(\frac{1}{2}\right)^{n-1}$

$a_9 = 3200\left(\frac{1}{2}\right)^{9-1}$

$= \boxed{12.5 \text{ or } \frac{25}{2}}$

25. $8, 24, 72, 216, 648, \ldots$

$a_n = 8(3)^{n-1}$

$a_9 = 8(3)^{9-1}$

$= \boxed{52{,}488}$

26. $a_4 = 54$ and $a_5 = 162$

$r = \frac{162}{54} = 3$

$a_4 = a_1(r)^{4-1}$

$54 = a_1(3)^3$

$54 = 27a_1$

$a_1 = 2$

$a_n = 2(3)^{n-1}$

$a_7 = 2(3)^{7-1}$

$= \boxed{1458}$

27. $a_5 = 13.5 \quad a_6 = 20.25$

$r = \frac{20.25}{13.5} = 1.5$

$a_7 = 1.5(20.25)$

$\boxed{a_7 = 30.375}$

28. $a_4 = -4 \quad a_6 = -100$

$a_1 = -4 \quad a_3 = -100$

$-100 = -4(r)^2$

$25 = r^2$

$r = \pm 5$

$\boxed{\begin{array}{l} a_7 = -100 \cdot 5 = -500 \\ \text{or } -100 \cdot -5 = 500 \end{array}}$

32. $S_6$ for $1 + 5 + 25 + 125 + \ldots$

$S_6 = 1\left(\frac{1-(5)^6}{1-5}\right) = \boxed{3906}$

33. $S_8$ for $10+1+\frac{1}{10}+\frac{1}{100}+\ldots$

$$S_8 = 10\left(\frac{1-\left(\frac{1}{10}\right)^8}{1-\frac{1}{10}}\right)$$

$$= 10\left(\frac{1-\frac{1}{100000000}}{\frac{9}{10}}\right)$$

$$= 10\left(\frac{\frac{99999999}{100000000}}{\frac{9}{10}}\right)$$

$$= 1\cancel{0}\left(\frac{99999999}{10000000\cancel{0}}\right)\left(\frac{\cancel{10}}{9}\right)$$

$$= \boxed{\frac{11111111}{1000000} = 11.111111}$$

34. $\sum_{k=1}^{6} -1\left(\frac{1}{3}\right)^{k-1}$

$$S_6 = -1\left(\frac{1-\left(\frac{1}{3}\right)^6}{1-\frac{1}{3}}\right)$$

$$= -1\left(\frac{1-\frac{1}{729}}{\frac{2}{3}}\right)$$

$$= -1\left(\frac{\frac{728}{729}}{\frac{2}{3}}\right)$$

$$= -1\left(\frac{728}{729}\right)\left(\frac{3}{2}\right) = \boxed{\frac{-364}{243}}$$

35. $\sum_{k=1}^{7} 8(10)^{k-1}$

$$S_7 = 8\left(\frac{1-(10)^7}{1-10}\right)$$

$$= 8\left(\frac{-9999999}{-9}\right)$$

$$= 8{,}888{,}888$$

36. $S_6 = 2\left(\frac{1-(2)^6}{1-2}\right)$
$= 2\left(\frac{-63}{-1}\right)$
$= 126$ ancestors
$S_{12} = 2\left(\frac{1-(2)^{12}}{1-2}\right)$
$= 2\left(\frac{-4095}{-1}\right)$
$= 8190$ ancestors
b. The initial value become 1 instead of 2.

37. $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \ldots$
a. $a_n = \frac{1}{16}(2)^{n-1}$
b. $a_{10} = \frac{1}{16}(2)^{10-1} = 32$
c. $S_{10} = \frac{1}{16}\left(\frac{1-(2)^{10}}{1-2}\right)$
$= 63.9375$ or $\frac{1023}{16}$

38. $4, 0.4, 0.04, 0.004, \ldots$
a. $a_n = 4(.1)^{n-1}$
b. $a_{10} = 4(.1)^{10-1} = 4 \times 10^{-9}$
c. $S_{10} = 4\left(\frac{1-.1^{10}}{1-.1}\right)$
$= 4.444444444$

39. $8, 16, 32, 64, \ldots$
$a_n = 8(2)^{n-1}$
$a_{10} = 8(2)^{10-1} = 4096$
$S_{10} = 8\left(\frac{1-2^{10}}{1-2}\right) = 8184$

41. $162, -54, 18, -6$
a. $a_n = 162(-\frac{1}{3})^{n-1}$
b. $a_{10} = 162(-\frac{1}{3})^{10-1} = -\frac{2}{243}$
c. $S_{10} = 162\left(\frac{1-(-\frac{1}{3})^{10}}{1-(-\frac{1}{3})}\right)$
$= 162\left(\frac{1-\frac{1}{59049}}{\frac{4}{3}}\right)$
$= 162\left(\frac{\frac{59048}{59049}}{\frac{4}{3}}\right)$
$= 162\left(\frac{59048}{59049}\right)\left(\frac{3}{4}\right)$
$= \boxed{\frac{29524}{243}}$

40. $-22, -11, -\frac{11}{2}, -\frac{11}{4}, \ldots$
a. $a_n = -22(\frac{1}{2})^{n-1}$
b. $a_{10} = -22(\frac{1}{2})^{10-1} = -\frac{11}{256}$
c. $S_{10} = -22\left(\frac{1-(\frac{1}{2})^{10}}{1-\frac{1}{2}}\right)$
$= -22\left(\frac{\frac{1023}{1024}}{\frac{1}{2}}\right)$
$= -22\left(\frac{1023}{1024}\right)\left(\frac{2}{1}\right)$
$= \boxed{\frac{-11253}{256}}$

42. $12.5, 62.5, 312.5, 1562.5, \ldots$
a. $a_n = 12.5(5)^{n-1}$
b. $a_{10} = 12.5(5)^{10-1} = 24414062.5$
c. $S_{10} = 12.5\left(\frac{1-5^{10}}{1-5}\right)$
$= 30{,}517{,}575$

44. $a_n = .01(2)^{n-1}$

$a_{18} = .01(2)^{18-1}$

$= 1310.72$

$a_{21} = .01(2)^{21-1}$

$= 10,485.76$

$S_{18} = .01\left(\frac{1-2^{18}}{1-2}\right)$

$= 2621.43$

$S_{22} = .01\left(\frac{1-2^{22}}{1-2}\right)$

$= 41,943.03$

45. $S_{10} = 1\left(\frac{1-5^{10}}{1-5}\right)$

$= 2,441,406$ emails

46. $a_n = 1\left(\frac{8}{9}\right)^{n-1}$

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | .8 | .70 | .62 | .55 | .49 |

$6^{th}$ iteration

49. $a_1 = 60 \quad a_3 = 9.6$

$9.6 = 60(r)^{3-1}$

$.16 = r^2$

$r = .4$

a. $60(.4) = \$24$ million

b. decreased By 60%

c.

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 60 | 24 | 9.6 | 3.84 | 1.536 | .61 |

In week 6, the sales would be less than \$1 million.

d. $S_8 = 60\left(\frac{1-(.4)^8}{1-.4}\right)$

$= \$99.93$ million