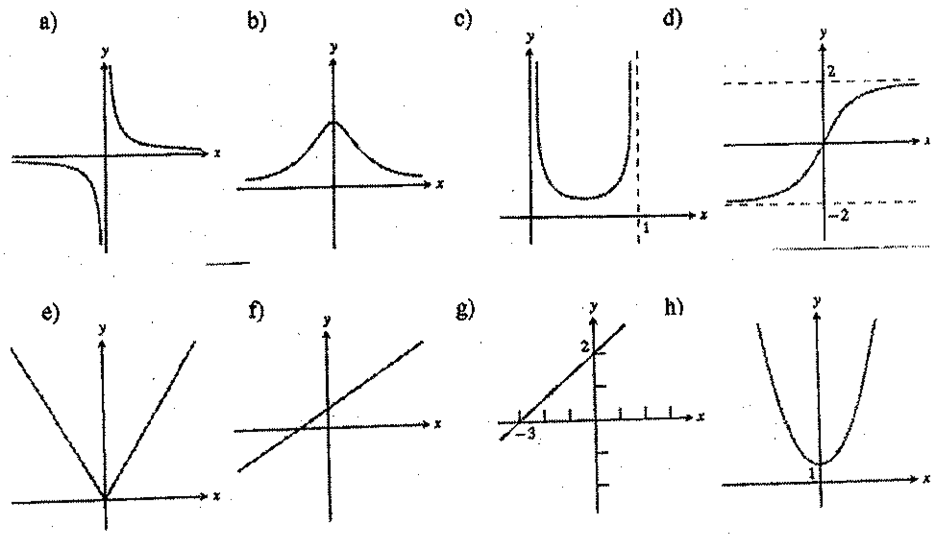
Honors GSE Algebra II — Name ______________________

Unit 3 – Polynomial Functions — WS – Inverses of Functions

1. Using the horizontal line test, determine if the functions are one-to-one. Write 'one-to-one' or 'not one-to-one' in the space provided.



a. one-to-one

b. not one-to-one

c. not one-to-one

d. one-to-one

e. not one-to-one

f. one-to-one

g. one-to-one

h. not one-to-one

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Find an equation for the inverse of each function, state the domain and range for each, and determine if the function is one-to-one.

D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

2. $f(x)=8x+3$

$x=8y+3$

$x-3=8y$

$\boxed{f^{-1}(x)=\frac{1}{8}x-\frac{3}{8}}$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$ one-to-one

D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

3. $f(x)=4(2x+3)$

$x=4(2y+3)$

$\frac{x}{4}=2y+3$

$\frac{x}{4}-3=2y$

$\boxed{f^{-1}(x)=\frac{x}{8}-\frac{3}{2}}$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$ one-to-one

4. $f(x)=-5-x-2x^2 = -2x^2-x-5$

$h=-\frac{1}{4}$

$k=\frac{-39}{8}$

D: $(-\infty,\infty)$

R: $(-\infty,-\frac{39}{8}]$

$f(x)=-2(x+\frac{1}{4})^2-\frac{39}{8}$

$x=-2(y+\frac{1}{4})^2-\frac{39}{8}$

$x+\frac{39}{8}=-2(y+\frac{1}{4})^2$

$-\frac{1}{2}(x+\frac{39}{8})=(y+\frac{1}{4})^2$

$\pm\sqrt{-\frac{1}{2}(x+\frac{39}{8})}=y+\frac{1}{4}$

$\boxed{f^{-1}(x)=\pm\sqrt{-\frac{1}{2}(x+\frac{39}{8})}-\frac{1}{4}}$ D: $(-\infty,-\frac{39}{8}]$ R: $(-\infty,\infty)$

not 1-to-1

5. $f(x)=x^2+7,\quad x\ge 0$

D: $[0,\infty)$ R: $[7,\infty)$

$x=y^2+7$

$x-7=y^2$

$\boxed{f^{-1}(x)=\sqrt{x-7}\quad x\ge 7}$ D: $[7,\infty)$ R: $[0,\infty)$ one-to-one

6. $f(x)=\frac{-x-8}{4}$

D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

$x=\frac{-y-8}{4}$

$4x=-y-8$

$4x+8=-y$

$\boxed{f^{-1}(x)=-4x-8}$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$ 1-to-1

7. $y=x^2-2x-35,\quad x\ge 1$, $y\ge -36$

D: $[1,\infty)$ R: $[36,\infty)$

$h=1$

$k=-36$

$y=(x-1)^2-36$

$x=(y-1)^2-36$

$x+36=(y-1)^2$

$\sqrt{x+36}=y-1$

$\boxed{f^{-1}(x)=\sqrt{x+36}+1}$ D: $[-36,\infty)$ R: $[1,\infty)$ 1-to-1

8. $f(x)=(x-4)^2$

D: $(-\infty,\infty)$ R: $[0,\infty)$

$x=(y-4)^2$

$\pm\sqrt{x}=y-4$

$\boxed{f^{-1}(x)=\pm\sqrt{x}+4}$ D: $[0,\infty)$ R: $(-\infty,\infty)$ not 1-to-1

9. $f(x)=2x^2+5x-3,\quad x\ge -1.25$

$h=-\frac{5}{4}$

$k=-\frac{49}{8}$

D: $[-\frac{5}{4},\infty)$

R: $[-\frac{49}{8},\infty)$

$y=2(x+\frac{5}{4})^2-\frac{49}{8}$

$x=2(y+\frac{5}{4})^2-\frac{49}{8}$

$x+\frac{49}{8}=2(y+\frac{5}{4})^2$

$\frac{1}{2}(x+\frac{49}{8})=(y+\frac{5}{4})^2$

$\sqrt{\frac{1}{2}(x+\frac{49}{8})}=y+\frac{5}{4}$

$\boxed{f^{-1}(x)=\sqrt{\frac{1}{2}(x+\frac{49}{8})}-\frac{5}{4}}$ D: $[-\frac{49}{8},\infty)$ R: $[-\frac{5}{4},\infty)$ 1-to-1

Determine by composition whether each pair of equations are inverses.

10. $f(x)=\sqrt{x}-9$ and $g(x)=x^2+9, x\geq 0$

$$f(g(x))=\sqrt{x^2+9}-9$$

$f(g(x))\neq x \therefore$ not inverses

11. $f(x)=\dfrac{2}{x-2}$ and $g(x)=\dfrac{x+2}{-2}$

$$f(g(x))=\frac{2}{\left(\frac{x+2}{-2}\right)-2}=\frac{2}{\frac{x+2+4}{-2}}=\frac{2}{\frac{x+6}{-2}}$$

$$=\frac{2}{1}\cdot\frac{-2}{x+6}=\frac{-4}{x+6}$$

$f(g(x))\neq x \therefore$ not inverses

12. $f(x)=\dfrac{x^2}{4}-1$ for $x\geq -1$ and $g(x)=\pm 2\sqrt{x+1}$

$$f(g(x))=\frac{(\pm 2\sqrt{x+1})^2}{4}-1$$

$$=\frac{4(x+1)}{4}-1$$

$$=x+1-1$$

$$=x$$

$$g(f(x))=\pm 2\sqrt{\left(\frac{x^2}{4}-1\right)+1}$$

$$=\pm 2\sqrt{\frac{x^2}{4}}$$

$$=\pm 2\left(\frac{x}{2}\right)$$

$$=\pm x$$

No, $g(f(x))\neq x \therefore$ $f(x)$ and $g(x)$ are not inverse functions

13. $f(x)=\dfrac{9}{5}x+32$ and $g(x)=\dfrac{5}{9}(x-32)$

$$f(g(x))=\frac{9}{5}\left(\frac{5}{9}(x-32)\right)+32$$

$$=x-32+32$$

$$=x$$

$$g(f(x))=\frac{5}{9}\left(\left(\frac{9}{5}x+32\right)-32\right)$$

$$=\frac{5}{9}\left(\frac{9}{5}x\right)$$

$$=x$$

$f(g(x))=g(f(x))=x$

$\therefore$ Yes, inverses