Honors Algebra II

Homework – Inverses of Rational Functions

Name: AK

A. Find the inverse of each function. Show your work.

B. Find the domains and ranges of the original function and the inverse function.

1. $y=-2x+5$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

$x=-2y+5$

$x-5=-2y$

$y^{-1}=-\frac{1}{2}x+\frac{5}{2}$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

2. $y=\frac{1}{3}x-2$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

$x=\frac{1}{3}y-2$

$x+2=\frac{1}{3}y$

$y^{-1}=3x+6$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

3. $y=\frac{1}{x}+6$ D: $\{x|x\neq 0\}$ R: $\{y|y\neq 6\}$

$x=\frac{1}{y}+6$

$x-6=\frac{1}{y}$

$y^{-1}=\frac{1}{x-6}$ D: $\{x|x\neq 6\}$ R: $\{y|y\neq 0\}$

4. $y=\frac{-1}{x+5}$ D: $\{x|x\neq -5\}$ R: $\{y|y\neq 0\}$

$x=\frac{-1}{y+5}$

$y+5=\frac{-1}{x}$

$y^{-1}=\frac{-1}{x}-5$ D: $\{x|x\neq 0\}$ R: $\{y|y\neq -5\}$

5. $y=\frac{5}{x-6}$ D: $\{x|x\neq 6\}$ R: $\{y|y\neq 0\}$

$x=\frac{5}{y-6}$

$y-6=\frac{5}{x}$

$y^{-1}=\frac{5}{x}+6$ D: $\{x|x\neq 0\}$ R: $\{y|y\neq 6\}$

6. $y=\frac{-2}{x-4}-5$ D: $\{x|x\neq 4\}$ R: $\{y|y\neq -5\}$

$x=\frac{-2}{y-4}-5$

$x+5=\frac{-2}{y-4}$

$y-4=\frac{-2}{x+5}$

$y^{-1}=\frac{-2}{x+5}+4$ D: $\{x|x\neq -5\}$ R: $\{y|y\neq 4\}$

7. $y=\frac{4-x}{2x+3}$ D: $\{x|x\neq -\frac{3}{2}\}$ R: $\{y|y\neq -\frac{1}{2}\}$

$y=\frac{-x+4}{2x+3}$

$\left(x=\frac{-y+4}{2y+3}\right)^{(2y+3)}$

$2xy+3x=-y+4$

$2xy+y=-3x+4$

$y(2x+1)=-3x+4$

$y^{-1}=\frac{-3x+4}{2x+1}$

D: $\{x|x\neq -\frac{1}{2}\}$

R: $\{y|y\neq -\frac{3}{2}\}$

8. $y=\frac{5x-6}{2x+7}$ D: $\{x|x\neq -\frac{7}{2}\}$ R: $\{y|y\neq \frac{5}{2}\}$

$x=\frac{5y-6}{2y+7}$

$2xy+7x=5y-6$

$2xy-5y=-7x-6$

$y(2x-5)=-7x-6$

$y^{-1}=\frac{-7x-6}{2x-5}$

D: $\{x|x\neq \frac{5}{2}\}$

R: $\{y|y\neq -\frac{7}{2}\}$

Verify that f(x) and g(x) are inverse functions. Show your work.

9. $f(x)=\frac{3x+1}{x-5}$, $g(x)=\frac{5x+1}{x-3}$

$$f(g(x))=\frac{3\left(\frac{5x+1}{x-3}\right)+1}{\left(\frac{5x+1}{x-3}\right)-5}=\frac{\frac{15x+3+x-3}{x-3}}{\frac{5x+1-5x+15}{x-3}}$$

$$=\frac{\frac{16x}{x-3}}{\frac{16}{x-3}}=\frac{16x}{x-3}\cdot\frac{x-3}{16}=x$$

$$g(f(x))=\frac{5\left(\frac{3x+1}{x-5}\right)+1}{\left(\frac{3x+1}{x-5}\right)-3}=\frac{\frac{15x+5+x-5}{x-5}}{\frac{3x+1-3x+15}{x-5}}$$

$$=\frac{\frac{16x}{x-5}}{\frac{16}{x-5}}=\frac{16x}{x-5}\cdot\frac{x-5}{16}=x$$

$f(g(x))=g(f(x))=x$, so $f(x)$ and $g(x)$ are inverses.

10. $f(x)=\frac{1}{x-4}+3$, $g(x)=\frac{4x-11}{x-3}$

$$f(g(x))=\frac{1}{\left(\frac{4x-11}{x-3}\right)-4}+3=\frac{1}{\frac{4x-11-4x+12}{x-3}}+3$$

$$=\frac{1}{\frac{1}{x-3}}+3=1\cdot\frac{x-3}{1}+3=x$$

$$g(f(x))=\frac{4\left(\frac{1}{x-4}+3\right)-11}{\left(\frac{1}{x-4}+3\right)-3}=\frac{\frac{4}{x-4}+12-11}{\frac{1}{x-4}}$$

$$=\frac{\frac{4}{x-4}+1}{\frac{1}{x-4}}=\frac{\frac{4+x-4}{x-4}}{\frac{1}{x-4}}=\frac{\frac{x}{x-4}}{\frac{1}{x-4}}=\frac{x}{x-4}\cdot\frac{x-4}{1}=x$$

$f(g(x))=g(f(x))=x$, so $f(x)$ and $g(x)$ are inverses.

For the equation below, sketch the original function and find then domain and range. Then sketch the inverse and find the domain and range.

11. $f(x)=\frac{-1}{x-2}+3$

Domain: $\{x\mid x\neq 2\}$

Range: $\{y\mid y\neq 3\}$

$x=\frac{-1}{y-2}+3$

$x-3=\frac{-1}{y-2}$

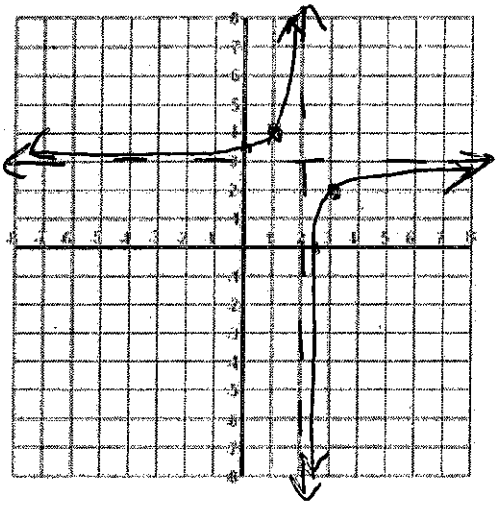
$y-2=\frac{-1}{x-3}$

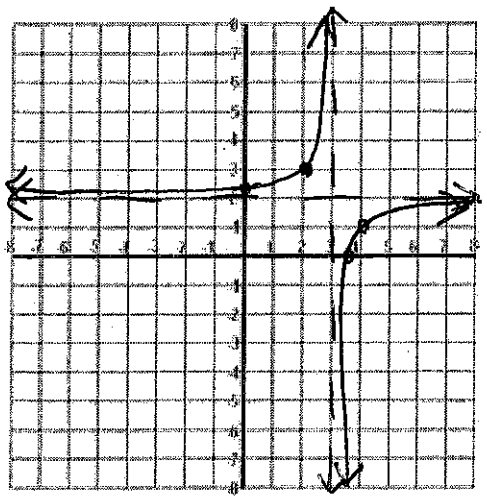
$y=\frac{-1}{x-3}+2$

$f^{-1}(x)=\frac{-1}{x-3}+2$

Domain: $\{x\mid x\neq 3\}$

Range: $\{y\mid y\neq 2\}$





$0=\frac{-1}{x-2}+3$

$-3=\frac{-1}{x-2}$

$-3x+6=-1$

$-3x=-7$

$x=\frac{7}{3}$