

Part I: Factor the following expressions.

1.  $7x^2 - 31x - 20$

2.  $28k^2 - 63$

3.  $7x^2 - 45x - 28$

4.  $2b^2 + 17bc + 21c^2$

5.  $121x^2 - 49y^2$

6.  $72x^3 + 36x^2 - 32x - 16$

Part II: Solve the following quadratics by factoring.

1.  $x^2 - 2x + 1 = 0$

2.  $10n^2 - 35 = 65n$

3.  $3x^2 - 8x = 16$

4.  $7a^2 + 32 = 7 - 40a$

5.  $3x(x - 1) = x^2$

6.  $42x^2 - 69x + 20 = 7x^2 - 8$

Part III: Solve the following quadratics by square-roots.

1.  $5x^2 + 16 = -44$

2.  $2(n - 2)^2 - 4 = 28$

3.  $-3(x - 3)^2 = 24$

Part IV: Solve the following quadratics by quadratic formula.

1.  $2n^2 - n - 4 = 2$

2.  $8n^2 - 4n = 18$

3.  $b^2 - 4b - 14 = -2$

4.  $10x^2 + 9 = x$

5.  $3a^2 = 6a - 3$

6.  $8a^2 + 7a = a - 5$

Part V: Solve the following quadratics by completing the square.

1.  $2n^2 - n - 4 = 2$

2.  $8n^2 - 4n = 18$

3.  $8a^2 + 7a = a - 5$

Part VI: Solve the following quadratic inequalities.

1.  $x^2 + 4x + 3 \leq 0$

2.  $5x^2 + 10 \geq 27x$

3.  $9x^2 + 6x + 1 \leq 0$

4.  $10x^2 - 4 > x$

5.  $9x^2 + 31x < -12$

6.  $4x^2 + 4x + 1 > 0$

Part VII: Graph the following quadratic equations and find the characteristics.

(Vertex, Axis of Symmetry, Domain, Range, Extrema, Intervals of Increase/Decrease, Roots, Y-Intercept, and End Behavior)

1.  $f(x) = x^2 + 4x + 3$

2.  $g(x) = -2(x + 3)^2 + 10$

3.  $h(x) = -2x^2 + 11x + 21$



Part I:

$$\begin{aligned} 1. & 7x^2 - 31x - 20 \\ & (7x^2 - 35x) + (4x - 20) \quad \begin{array}{r} -140 \\ -35 \times 4 \\ -31 \end{array} \\ & 7x(x-5) + 4(x-5) \\ & \boxed{(7x+4)(x-5)} \end{aligned}$$

$$\begin{aligned} 2. & 28k^2 - 63 \\ & 7(4k^2 - 9) \\ & \boxed{7(2k+3)(2k-3)} \end{aligned}$$

$$\begin{aligned} 3. & 7x^2 - 45x - 28 \\ & (7x^2 - 49x) + (4x - 28) \quad \begin{array}{r} -196 \\ -49 \times 4 \\ -45 \end{array} \\ & 7x(x-7) + 4(x-7) \\ & \boxed{(7x+4)(x-7)} \end{aligned}$$

$$\begin{aligned} 4. & 2b^2 + 17bc + 21c^2 \\ & (2b^2 + 14bc) + (3bc + 21c^2) \quad \begin{array}{r} 42 \\ 14 \times 3 \\ 17 \end{array} \\ & 2b(b+7c) + 3c(b+7c) \\ & \boxed{(2b+3c)(b+7c)} \end{aligned}$$

$$\begin{aligned} 5. & 121x^2 - 49y^2 \\ & \boxed{(11x-7y)(11x+7y)} \end{aligned}$$

$$\begin{aligned} 6. & (72x^3 + 36x^2 - 32x - 16) \\ & 84(18x^3 + 9x^2 - 8x - 4) \\ & 9x^2(2x+1) - 4(2x+1) \\ & (9x^2 - 4)(2x+1) \\ & \boxed{4(3x+2)(3x-2)(2x+1)} \end{aligned}$$

## Part II

1.  $x^2 - 2x + 1 = 0$

$$(x-1)(x-1) = 0$$

$$(x-1)^2 = 0$$

$$x-1 = 0$$

$$\boxed{x = 1}$$

5.  $3x(x-1) = x^2$

$$3x^2 - 3x = x^2$$

$$2x^2 - 3x = 0$$

$$x(2x-3) = 0$$

$$\boxed{x = 0}$$

$$2x-3 = 0$$

$$\boxed{x = \frac{3}{2}}$$

2.  $10n^2 - 35 = 65n$

$$10n^2 - 65n - 35 = 0$$

$$5(2n^2 - 13n - 7) = 0$$

$$5(2n^2 - 14n + n - 7) = 0$$

$$5(2n(n-7) + 1(n-7)) = 0$$

$$5(2n+1)(n-7) = 0$$

$$2n+1 = 0 \quad n-7 = 0$$

$$\boxed{n = -\frac{1}{2}}$$

$$\boxed{n = 7}$$

6.  $42x^2 - 69x + 20 = 7x^2 - 8$

$$-7x^2 + 8 \quad -7x^2 + 8$$

$$35x^2 - 69x + 28 = 0$$

$$(35x^2 - 49x) - (20x - 28) = 0$$

$$7x(5x-7) - 4(5x-7) = 0$$

$$(7x-4)(5x-7) = 0$$

$$7x-4 = 0$$

$$5x-7 = 0$$

$$\boxed{x = \frac{4}{7}}$$

$$\boxed{x = \frac{7}{5}}$$

3.  $3x^2 - 8x = 16$

$$3x^2 - 8x - 16 = 0$$

$$(3x^2 - 12x) + (4x - 16) = 0$$

$$3x(x-4) + 4(x-4) = 0$$

$$(3x+4)(x-4) = 0$$

$$3x+4 = 0 \quad x-4 = 0$$

$$\boxed{x = -\frac{4}{3}}$$

$$\boxed{x = 4}$$

4.  $7a^2 + 32 = 7 - 40a$

$$+40a - 7 \quad -7 + 40a$$

$$7a^2 + 40a + 25 = 0$$

$$(7a^2 + 5a) + (35a + 25) = 0$$

$$a(7a+5) + 5(7a+5) = 0$$

$$(a+5)(7a+5) = 0$$

$$a+5 = 0$$

$$7a+5 = 0$$

$$\boxed{a = -5}$$

$$\boxed{a = -\frac{5}{7}}$$

### Part III

$$1. \begin{aligned} 5x^2 + 16 &= -44 \\ 5x^2 &= -60 \\ x^2 &= -12 \\ x &= \pm 2i\sqrt{3} \end{aligned}$$

$$2. \begin{aligned} 2(n-2)^2 - 4 &= 28 \\ 2(n-2)^2 &= 32 \\ (n-2)^2 &= 16 \\ n-2 &= \pm 4 \\ n &= 2+4 \quad n = 2-4 \\ n &= 6 \quad n = -2 \end{aligned}$$

$$3. \begin{aligned} -3(x-3)^2 &= 24 \\ (x-3)^2 &= -8 \\ x-3 &= \pm 2i\sqrt{2} \\ x &= 3 \pm 2i\sqrt{2} \end{aligned}$$

### Part IV

$$1. \begin{aligned} 2n^2 - n - 4 &= 2 \\ 2n^2 - n - 6 &= 0 \\ (-1)^2 - 4(2)(-6) &= 49 \\ \frac{1 \pm \sqrt{49}}{2(2)} &= \frac{1 \pm 7}{4} \\ x &= 2, -\frac{3}{2} \end{aligned}$$

$$2. \begin{aligned} 8n^2 - 4n &= 18 \\ 8n^2 - 4n - 18 &= 0 \\ 2(4n^2 - 2n - 9) &= 0 \\ (-2)^2 - 4(4)(-9) &= 148 \\ \frac{2 \pm \sqrt{148}}{2(4)} &= \frac{2 \pm 2\sqrt{37}}{8} \\ x &= \frac{1 \pm \sqrt{37}}{4} \end{aligned}$$

$$3. \begin{aligned} b^2 - 4b - 14 &= -2 \\ b^2 - 4b - 12 &= 0 \\ (-4)^2 - 4(1)(-12) &= 64 \\ \frac{4 \pm \sqrt{64}}{2(1)} &= \frac{4 \pm 8}{2} \\ x &= -2, 6 \end{aligned}$$

$$4. \begin{aligned} 10x^2 + 9 &= x \\ 10x^2 - x + 9 &= 0 \\ (-1)^2 - 4(10)(9) &= -359 \\ \frac{1 \pm \sqrt{-359}}{2(10)} &= \frac{1 \pm i\sqrt{359}}{20} \\ x &= \frac{1 \pm i\sqrt{359}}{20} \end{aligned}$$

$$5. \begin{aligned} 3a^2 &= 6a - 3 \\ 3a^2 - 6a + 3 &= 0 \\ 3(a^2 - 2a + 1) &= 0 \\ (-2)^2 - 4(1)(1) &= 0 \\ \frac{2 \pm \sqrt{0}}{2(1)} &= \frac{2}{2} \\ x &= 1 \end{aligned}$$

$$6. \begin{aligned} 8a^2 + 7a &= a - 5 \\ -a + 5 \quad -a + 5 \\ 8a^2 + 6a + 5 &= 0 \\ (6)^2 - 4(8)(5) &= -124 \\ \frac{-6 \pm \sqrt{-124}}{2(8)} &= \frac{-6 \pm 2i\sqrt{31}}{16} \\ x &= \frac{-3 \pm i\sqrt{31}}{8} \end{aligned}$$

## Part V

1.  $2n^2 - n - 4 = 2$

$$2n^2 - n = 6$$

$$\frac{2}{2} + \frac{1}{2} = \frac{6}{2}$$

$$n^2 - \frac{n}{2} = 3$$

$$+ \frac{1}{16} \quad + \frac{1}{16}$$

$$n^2 - \frac{n}{2} + \frac{1}{16} = \frac{49}{16}$$

$$(n - \frac{1}{4})^2 = \frac{49}{16}$$

$$n - \frac{1}{4} = \pm \frac{7}{4}$$

$$n = \frac{1}{4} \pm \frac{7}{4}$$

$$n = 2, -\frac{3}{2}$$

2.  $8n^2 - 4n = 18$

$$\frac{8}{8} \quad \frac{8}{8} \quad \frac{8}{8}$$

$$n^2 - \frac{1}{2}n = \frac{9}{4}$$

$$+ \frac{1}{16} \quad + \frac{1}{16}$$

$$n^2 - \frac{1}{2}n + \frac{1}{16} = \frac{37}{16}$$

$$(n - \frac{1}{4})^2 = \frac{37}{16}$$

$$n - \frac{1}{4} = \pm \frac{\sqrt{37}}{4}$$

$$n = \frac{1 \pm \sqrt{37}}{4}$$

3.  $8a^2 + 7a = a - 5$

$$8a^2 + 6a = -5$$

$$\frac{8}{8} \quad \frac{8}{8} \quad \frac{8}{8}$$

$$a^2 + \frac{3}{4}a = -\frac{5}{8}$$

$$+ \frac{9}{64} \quad + \frac{9}{64}$$

$$(a + \frac{3}{8})^2 = -\frac{31}{64}$$

$$a + \frac{3}{8} = \pm \frac{\sqrt{31}}{8}$$

$$a = -\frac{3}{8} \pm \frac{\sqrt{31}}{8}$$

## Part VI

1.  $x^2 + 4x + 3 \leq 0$

$$(x+3)(x+1) = 0$$

$$x = -3, -1$$



$$[-3, -1]$$

2.  $5x^2 + 10 \geq 27x$

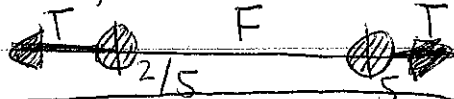
$$5x^2 - 27x + 10 \geq 0$$

$$5x^2 - 25x - 2x + 10 = 0$$

$$5x(x-5) - 2(x-5) = 0$$

$$(5x-2)(x-5) = 0$$

$$x = \frac{2}{5}, 5$$



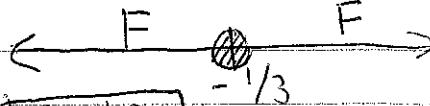
$$(-\infty, \frac{2}{5}] \cup [5, \infty)$$

3.  $9x^2 + 6x + 1 \leq 0$

$$(3x+1)(3x+1) \leq 0$$

$$(3x+1)^2 \leq 0$$

$$x = -\frac{1}{3}$$



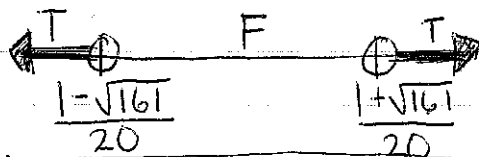
$$\{-\frac{1}{3}\}$$

$$4. \quad 10x^2 - 4 > x$$

$$10x^2 - x - 4 > 0$$

$$(-1)^2 - 4(10)(-4) = 161$$

$$\frac{1 \pm \sqrt{161}}{2(10)} = \frac{1 \pm \sqrt{161}}{20}$$



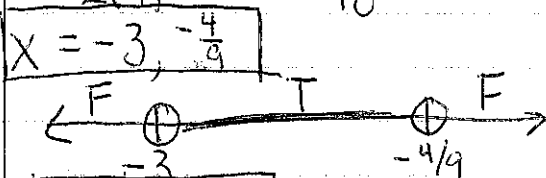
$$\boxed{(-\infty, \frac{1 - \sqrt{161}}{20}) \cup (\frac{1 + \sqrt{161}}{20}, \infty)}$$

$$5. \quad 9x^2 + 31x < -12$$

$$9x^2 + 31x + 12 < 0$$

$$(31)^2 - 4(9)(12) = 529$$

$$\frac{-31 \pm \sqrt{529}}{2(9)} = \frac{-31 \pm 23}{18}$$

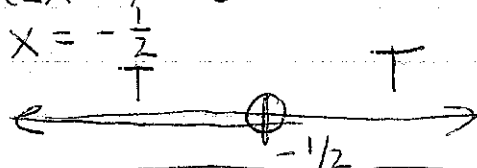


$$\boxed{(-3, -\frac{4}{9})}$$

$$6. \quad 4x^2 + 4x + 1 > 0$$

$$(2x+1)(2x+1) > 0$$

$$(2x+1)^2 > 0$$



$$\boxed{(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)}$$

## Part VII

1.  $f(x) = x^2 + 4x + 3$   
 $= (x+3)(x+1)$   
 $\frac{-3+(-1)}{2} = -2$

Vertex:  $(-2, -1)$

AoS:  $x = -2$

D:  $(-\infty, \infty)$

R:  $[-1, \infty)$

Extrema: min of  $y = -1$   
 at  $(-2, -1)$

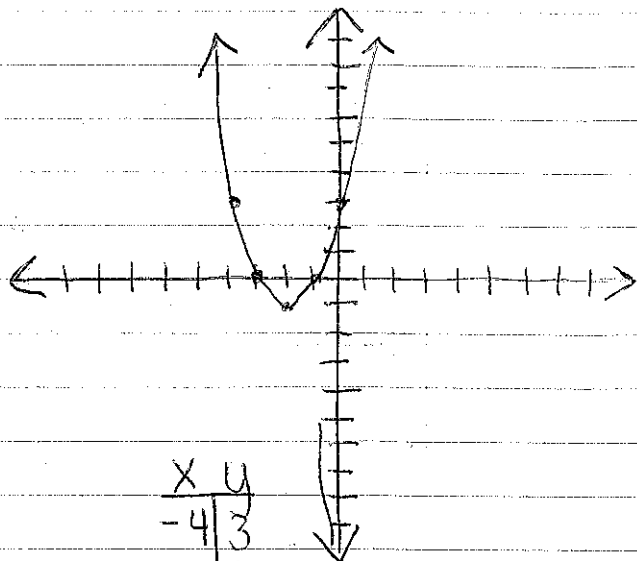
Inc:  $(-2, \infty)$  Dec:  $(-\infty, -2)$

Roots:  $x = -3, -1$

Y-int:  $(0, 3)$

End: As  $x \rightarrow -\infty, f(x) \rightarrow \infty$

As  $x \rightarrow \infty, f(x) \rightarrow \infty$



x	y
-4	3
-3	0
-2	-1
-1	0
0	3

2.  $g(x) = -2(x+3)^2 + 10$

Vertex:  $(-3, 10)$

AoS:  $x = -3$

D:  $(-\infty, \infty)$

R:  $(-\infty, 10]$

Extrema: max of  $y = 10$   
 at  $(-3, 10)$

Inc:  $(-\infty, -3)$  Dec:  $(-3, \infty)$

Roots:  $-2(x+3)^2 + 10 = 0$

$$(x+3)^2 = 5$$

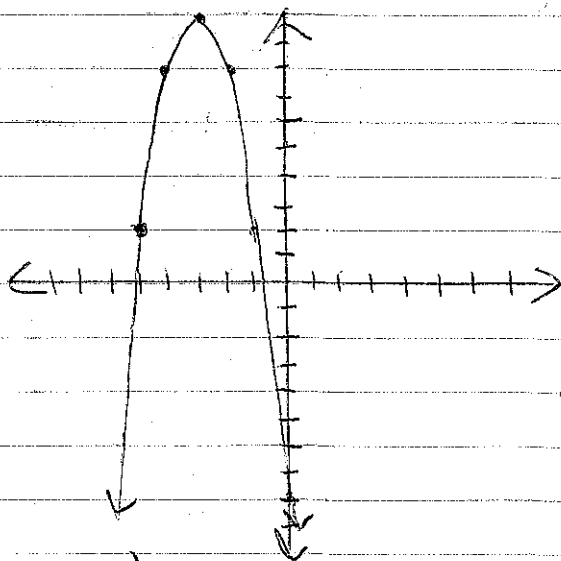
$$x+3 = \pm\sqrt{5}$$

$$x = -3 \pm\sqrt{5}$$

Y-int:  $(0, -8)$

End: As  $x \rightarrow -\infty, g(x) \rightarrow -\infty$

As  $x \rightarrow \infty, g(x) \rightarrow -\infty$



x	y
-5	2
-4	8
-3	10
-2	8
-1	2



3.  $h(x) = -2x^2 + 11x + 21$

$\frac{-11}{2(-2)} = \frac{11}{4}$

Vertex:  $(\frac{11}{4}, \frac{289}{8})$

AoS:  $x = \frac{11}{4}$

D:  $(-\infty, \infty)$

R:  $(-\infty, \frac{289}{8}]$

Extrema: max of  $y = \frac{289}{8}$   
at  $(\frac{11}{4}, \frac{289}{8})$

Inc:  $(-\infty, \frac{11}{4})$  Dec:  $(\frac{11}{4}, \infty)$

Roots:  $(11)^2 - 4(-2)(21) = 289$

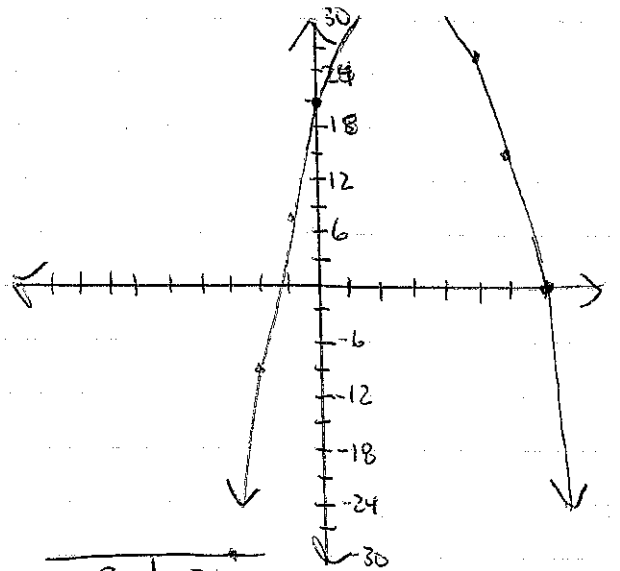
$\frac{-11 \pm \sqrt{289}}{-4} = \frac{-11 \pm 17}{-4}$

$x = 7, -\frac{3}{2}$

y-int:  $(0, 21)$

End: As  $x \rightarrow -\infty, h(x) \rightarrow -\infty$

As  $x \rightarrow \infty, h(x) \rightarrow -\infty$



-3	-30
-2	-9
-1	8
0	21
5	26
6	15
7	0
8	-19

