

Simplify Rational Exponents

Simplify each of the following rational exponent expressions. Do not use a calculator.

a. $9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = \boxed{3}$

b. $9^{\frac{5}{2}} = (3^2)^{\frac{5}{2}} = 3^5 = \boxed{243}$

c. $27^{\frac{-2}{3}} = (3^3)^{\frac{-2}{3}} = 3^{-2} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

d. $-2^{\frac{4}{2}} = -2^2 = \boxed{-4}$

e. $1,000,000^{\frac{1}{6}} = (10^6)^{\frac{1}{6}} = 10^1 = \boxed{10}$

f. $(-32)^{\frac{3}{5}} = ((-2)^5)^{\frac{3}{5}} = (-2)^3 = \boxed{-8}$

g. $64^{\frac{2}{3}} = (2^6)^{\frac{2}{3}} = 2^4 = \boxed{16}$
 or $(4^3)^{\frac{2}{3}} = 4^2 = \boxed{16}$

h. $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^1 = \boxed{5}$

i. $-81^{\frac{-3}{4}} = -(3^4)^{\frac{-3}{4}} = -(3)^{-3} = -\left(\frac{1}{3^3}\right) = \boxed{-\frac{1}{27}}$

j. $\sqrt{81} \cdot \sqrt[3]{8} = 81^{\frac{1}{2}} \cdot 8^{\frac{1}{3}} = (9^2)^{\frac{1}{2}} \cdot (2^3)^{\frac{1}{3}} = 9 \cdot 2 = \boxed{18}$

k. $16^{\frac{-1}{4}} = (2^4)^{\frac{-1}{4}} = 2^{-1} = \boxed{\frac{1}{2}}$

l. $(x^6)^{\frac{1}{2}} = \boxed{x^3}$

		Roots										
		0	1	2	3	4	5	6	7	8	9	10
Bases	0	?	0	0	0	0	0	0	0	0	0	0
	1	1	1	1	1	1	1	1	1	1	1	1
	2	1	2	4	8	16	32	64	128	256	512	1024
	3	1	3	9	27	81	243	729	2187	6561	19683	59049
	4	1	4	16	64	256	1024	4096	16384	65536	262144	*
	5	1	5	25	125	625	3125	15625	78125	390625	*	*
	6	1	6	36	216	1296	7776	46656	279936	*	*	*
	7	1	7	49	343	2401	16807	117649	823543	*	*	*
	8	1	8	64	512	4096	32768	262144	*	*	*	*
	9	1	9	81	729	6561	59049	531441	*	*	*	*
	10	1	10	100	1000	10000	100000	1000000	*	*	*	*

Using Properties of Rational Exponents

The properties of integer exponents can also be applied to rational exponents. Rewrite so final answer is using positive, rational exponents.

$$a. 9^{\frac{1}{4}} \cdot 9^{\frac{7}{4}} = 9^{\frac{1}{4} + \frac{7}{4}} = 9^{\frac{8}{4}} = 9^2 = \boxed{81}$$

$$b. 5^{\frac{1}{2}} \cdot 5^{\frac{1}{4}} = 5^{\frac{1}{2} + \frac{1}{4}} = \boxed{5^{\frac{3}{4}}}$$

$$c. 4^{\frac{1}{4}} \cdot 64^{\frac{1}{4}} = 4^{\frac{1}{4}} \cdot (4^3)^{\frac{1}{4}} = 4^{\frac{1}{4}} \cdot 4^{\frac{3}{4}} = 4^{\frac{1}{4} + \frac{3}{4}} = \boxed{4}$$

$$d. (8^{\frac{1}{2}} \cdot 5^{\frac{1}{3}})^2 = 8^{\frac{2}{2}} \cdot 5^{\frac{2}{3}} = \boxed{8 \cdot 5^{\frac{2}{3}}}$$

$$e. (2^4 \cdot 3^4)^{-\frac{1}{4}} = 2^{-1} \cdot 3^{-1} = \frac{1}{2} \cdot \frac{1}{3} = \boxed{\frac{1}{6}}$$

$$f. \frac{27^{\frac{1}{3}}}{27^{\frac{2}{3}}} = 27^{\frac{1}{3} - \frac{2}{3}} = 27^{-\frac{1}{3}} = (3^3)^{-\frac{1}{3}} = 3^{-1} = \boxed{\frac{1}{3}}$$

$$g. \frac{7}{7^{\frac{1}{3}}} = 7^{1 - \frac{1}{3}} = \boxed{7^{\frac{2}{3}}}$$

$$f. \left(\frac{12^{\frac{1}{3}}}{4^{\frac{1}{3}}} \right)^2 = \left(\frac{4^{\frac{1}{3}} \cdot 3^{\frac{1}{3}}}{4^{\frac{1}{3}}} \right)^2 = (3^{\frac{1}{3}})^2 = \boxed{3^{\frac{2}{3}}}$$

$$g. \left(\frac{8}{27} \right)^{\frac{1}{3}} = \left(\frac{2^3}{3^3} \right)^{\frac{1}{3}} = \boxed{\frac{2}{3}}$$

$$h. \frac{\sqrt[3]{3}}{\sqrt[3]{24}} = \frac{3^{\frac{1}{3}}}{(2^3 \cdot 3)^{\frac{1}{3}}} = \frac{3^{\frac{1}{3}}}{2 \cdot 3^{\frac{1}{3}}} = \boxed{\frac{1}{2}}$$

$$i. \frac{\sqrt{6}}{\sqrt[4]{6}} = \frac{6^{\frac{1}{2}}}{6^{\frac{1}{4}}} = 6^{\frac{1}{2} - \frac{1}{4}} = \boxed{6^{\frac{1}{4}}}$$

$$j. \frac{(36m^6)^{\frac{1}{2}}}{(6^2 m^6)^{\frac{1}{2}}} = \boxed{6m^3}$$

$$k. (64n^{12})^{-\frac{1}{6}} = (2^6 n^{12})^{-\frac{1}{6}} = 2^{-1} n^{-2} = \boxed{\frac{1}{2n^2}}$$

$$l. \left(x^3 y^{\frac{1}{3}} \right)^3 \cdot \sqrt[4]{x^4} = (x^3 y^{\frac{1}{3}})^3 \cdot (x^4)^{\frac{1}{4}} = x^9 y \cdot x = \boxed{x^{10} y}$$

$$m. (x^6 y^8)^{\frac{1}{2}} \cdot \sqrt[3]{y^6} = (x^6 y^8)^{\frac{1}{2}} \cdot (y^6)^{\frac{1}{3}} = x^3 y^4 \cdot y^2 = \boxed{x^3 y^6}$$