WS - Solving Rational Equations

1. $\left(\frac{3x}{x-2} = 1 + \frac{6}{x-2}\right)(x-2)$

$\frac{3x(x-2)}{x-2} = 1(x-2) + \frac{6(x-2)}{(x-2)}$

$3x = x - 2 + 6$

$2x = 4$

$x = 2$ $\quad x \neq 2$

no solution

2. $\left(\frac{4}{x} - \frac{1}{x+2} = \frac{2}{x}\right)x(x+2)$

$\frac{4(x)(x+2)}{x} - \frac{1(x)(x+2)}{(x+2)} = \frac{2(x)(x+2)}{x}$

$4(x+2) - 1(x) = 2(x+2)$

$4x + 8 - x = 2x + 4$

$3x + 8 = 2x + 4$

$\boxed{x = -4}$ ✓ $\quad x \neq 0, -2$

3. $\left[\frac{2}{x-10} - \frac{3}{x-2} = \frac{6}{(x-10)(x-2)}\right](x-10)(x-2)$

$\frac{2(x-10)(x-2)}{(x-10)} - \frac{3(x-10)(x-2)}{(x-2)} = \frac{6(x-10)(x-2)}{(x-10)(x-2)}$

$2(x-2) - 3(x-10) = 6$

$2x - 4 - 3x + 30 = 6$

$-x + 26 = 6$

$-x = -20$

$\boxed{x = 20}$ ✓ $\quad x \neq 2, 10$

4. $\left[\frac{3x}{x-2} + \frac{1}{x+2} = \frac{-4}{(x-2)(x+2)}\right](x-2)(x+2)$

$\frac{3x(x-2)(x+2)}{(x-2)} + \frac{1(x-2)(x+2)}{(x+2)} = \frac{-4(x-2)(x+2)}{(x-2)(x+2)}$

$3x(x+2)+1(x-2)=-4$

$3x^2+6x+x-2=-4$

$3x^2+7x+2=0$

$3x^2+6x+x+2=0$

$(3x+1)(x+2)=0$

$\boxed{x=-1/3, -2}$ $\quad x\neq 2, -2$

5. $\left(\frac{5x}{x-1} - 3 = \frac{2x+5}{x^2-1}\right)(x-1)(x+1)$

$\frac{5x(x-1)(x+1)}{(x-1)} - 3(x-1)(x+1) = \frac{(2x+5)(x-1)(x+1)}{(x-1)(x+1)}$

$5x^2+5x-3x^2+3=2x+5$

$2x^2+3x-2=0$

$2x^2+4x-1x-2=0$

$(2x-1)(x+2)=0$

$\boxed{x=1/2, -2}$ $\quad x\neq 1, -1$

6. $\left[\frac{2x}{x-2} - \frac{4x-1}{3x+2} = \frac{17x+4}{(x-2)(3x+2)}\right](x-2)(3x+2)$

$\frac{2x(x-2)(3x+2)}{x-2} - \frac{(4x-1)(x-2)(3x+2)}{(3x+2)} = \frac{(17x+4)(x-2)(3x+2)}{(x-2)(3x+2)}$

$6x^2+4x-(4x^2-9x+2)=17x+4$

$2x^2+13x-2=17x+4$

$2x^2-4x-6=0$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$\boxed{x=3, -1}$ $\quad x\neq 2, -\frac{2}{3}$

7. $\left(\frac{1}{x+1} + \frac{2}{x+2} = \frac{x+3}{4}\right)(x+1)(x+2)(4)$

$\frac{1(x+1)(x+2)(4)}{x+1} + \frac{2(x+1)(x+2)(4)}{x+2} = \frac{(x+3)(x+1)(x+2)(4)}{4}$

$4x+8+8x+8 = (x^2+5x+6)(x+1)$

$12x+16 = x^3+6x^2+11x+6$

$0 = x^3+6x^2-x-10$

From graphing calc. $\boxed{x = -5.88, -1.36, 1.25}$

$x \neq -1, -2$

8. $\left(\frac{x}{x-3} = \frac{1}{4x^2}\right)(4x^2)(x-3)$

$x(4x^2) = 1(x-3)$

$4x^3 = x-3$

$4x^3-x+3 = 0$ $\quad \pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

1 | 4 0 −1 3
4 4 3
4 4 3 | 6

−1 | 4 0 −1 3
−4 4 −3
4 −4 3 | 0

$0 = (x+1)(4x^2-4x+3)$

$(-4)^2-4(4)(3) = -32$ no real solutions

$\boxed{x = -1}$ $\quad x \neq 3, 0$

9. $\left[\frac{2}{x+3} - \frac{3}{-1(x-4)} = \frac{2x-12}{(x-4)(x+3)}\right](x-4)(x+3)$

$\frac{2(x-4)(x+3)}{x+3} + \frac{3(x-4)(x+3)}{x-4} = \frac{(2x-12)(x-4)(x+3)}{(x-4)(x+3)}$

$2x-8+3x+9 = 2x-12$

$5x+1 = 2x-12$

$3x = -13$

$\boxed{x = -\frac{13}{3}}$ $\quad x \neq -3, 4$

10. $\left[\frac{x}{x+4} - \frac{28}{(x+4)(x-3)} = \frac{1}{x-3}\right](x+4)(x-3)$

$\frac{x(x+4)(x-3)}{x+4} - \frac{28(x+4)(x-3)}{(x+4)(x-3)} = \frac{1(x+4)(x-3)}{x-3}$

$x^2-3x-28 = x+4$

$x^2-4x-32 = 0$

$(-4)^2-4(1)(-32) = 144$

$\frac{4 \pm \sqrt{144}}{2} = \frac{4 \pm 12}{2} = \boxed{8, -4}$ $\quad x \neq -4, 3$