Honors Advanced Algebra — Name ____________________

Transformations of Polynomials, WS 2

1. Use the polynomial $f(x) = -2x^3 + 5x$ to perform the following transformations to find a new polynomial. Then, describe the transformations.

a. $g(x) = -f(x)$ $\boxed{2x^3 - 5x}$ reflect over x-axis

b. $h(x) = f(x-3)$

$-2(x-3)^3 + 5(x-3)$

$-2(x^3 + 3x^2(-3) + 3x(-3)^2 + (-3)^3) + 5x - 15$

$-2(x^3 - 9x^2 + 27x - 27) + 5x - 15$

$-2x^3 + 18x^2 - 54x + 54 + 5x - 15$

$\boxed{-2x^3 + 18x^2 - 49x + 39}$

horizontal shift right 3

c. $m(x) = 3f(x) + 2$ $3(-2x^3 + 5x) + 2$

$\boxed{-6x^3 + 15x + 2}$

stretch by 3

vertical shift up 2

d. $z(x) = -f(x+1) - 4$

$-(-2(x+1)^3 + 5(x+1)) - 4$

$-(-2(x^3 + 3x^2 + 3x + 1) + 5x + 5) - 4$

$-(-2x^3 - 6x^2 - 6x - 2 + 5x + 5) - 4$

$-(-2x^3 - 6x^2 - x + 3) - 4$

$2x^3 + 6x^2 + x - 3 - 4$

$\boxed{2x^3 + 6x^2 + x - 7}$

reflect over x-axis, shift left 1, shift down 4

2. P(x) has the following characteristics. Fill out the missing characteristics. Use these to find the new characteristics after P(x) has been transformed. (Hint: it might help to sketch a rough graph)

| | | | |
|---|---|---|---|
| Leading Coefficient | -.1 | Absolute Maximum | (-1.3, 15.3) |
| Degree | 6 | Relative Maximum | (5, 0) |
| Domain | ℝ | Absolute Minimum | none |
| Range | $(-\infty, 15.3]$ | Relative Minimum | (3.2, -57.2) |
| X-Intercepts | (-2, 0) (1, 0) (5,0)mult.2 | Intervals of Increase | $(-\infty, -1.3)$ $(3.2, 5)$ |
| Y-Intercepts | (0, +10) | Intervals of Decrease | $(-1.3, 3.2)$ $(5, \infty)$ |
| End Behavior | As $x \to -\infty$, $P(x) \to -\infty$<br>As $x \to \infty$, $P(x) \to -\infty$ | | |

a. $T(x) = -P(x)$ Multiply all y-values by -1

| | | | |
|---|---|---|---|
| Leading Coefficient | .1 | Absolute Maximum | none |
| Degree | 6 | Relative Maximum | (3.2, 57.2) |
| Domain | ℝ | Absolute Minimum | (-1.3, -15.3) |
| Range | $[-15.3, \infty)$ | Relative Minimum | (5, 0) |
| X-Intercepts | (-2,0) (1,0) (5,0)mult.2 | Intervals of Increase | $(-1.3, 3.2)(5, \infty)$ |
| Y-Intercepts | (0, -10) | Intervals of Decrease | $(-\infty, -1.3)(3.2, 5)$ |
| End Behavior | As $x \to -\infty$, $T(x) \to \infty$<br>As $x \to \infty$, $T(x) \to \infty$ | | |

intervals switched from original

b. $H(x) = P(x) - 4$ subtract 4 from all y-values

| | | | |
|---|---|---|---|
| Leading Coefficient | -.1 | Absolute Maximum | (-1.3, 11.3) |
| Degree | 6 | Relative Maximum | (5, -4) |
| Domain | ℝ | Absolute Minimum | none |
| Range | (-∞, 11.3] | Relative Minimum | (3.2, -61.2) |
| X-Intercepts | can't be determined | Intervals of Increase | (-∞, -1.3) (3.2, 5) |
| Y-Intercepts | (0, 6) | Intervals of Decrease | (-1.3, 3.2) (5, ∞) |
| End Behavior | As x → -∞, H(x) → -∞<br>As x → ∞, H(x) → -∞ | | |

c. $M(x) = P(x - 5)$ shifts 5 right - add 5 to all x-values

| | | | |
|---|---|---|---|
| Leading Coefficient | -.1 | Absolute Maximum | (3.7, 15.3) |
| Degree | 6 | Relative Maximum | (10, 0) |
| Domain | ℝ | Absolute Minimum | none |
| Range | (-∞, 15.3] | Relative Minimum | (8.2, -57.2) |
| X-Intercepts | (3,0) (6,0) (10,0) mult. 2 | Intervals of Increase | (-∞, 3.7) (8.2, 10) |
| Y-Intercepts | can't be determined | Intervals of Decrease | (3.7, 8.2) (10, ∞) |
| End Behavior | As x → -∞, M(x) → -∞<br>As x → ∞, M(x) → -∞ | | |

d. $J(x) = 2P(x)$ multiply all y-values by 2

| | | | |
|---|---|---|---|
| Leading Coefficient | -.2 | Absolute Maximum | (-1.3, 30.6) |
| Degree | 6 | Relative Maximum | (5, 0) |
| Domain | ℝ | Absolute Minimum | none |
| Range | (-∞, 30.6] | Relative Minimum | (3.2, -114.4) |
| X-Intercepts | (-2,0) (1,0) (5,0) mult. 2 | Intervals of Increase | (-∞, -1.3) (3.2, 5) |
| Y-Intercepts | (0, 20) | Intervals of Decrease | (-1.3, 3.2) (5, ∞) |
| End Behavior | As x → -∞, J(x) → -∞<br>As x → ∞, J(x) → -∞ | | |

e. $W(x) = 3P(x) + 2$ multiply all y-values by 3 and add 2

| | | | |
|---|---|---|---|
| Leading Coefficient | -.3 | Absolute Maximum | (-1.3, 47.9) |
| Degree | 6 | Relative Maximum | (5, 2) |
| Domain | ℝ | Absolute Minimum | none |
| Range | (-∞, 47.9] | Relative Minimum | (3.2, -169.6) |
| X-Intercepts | can't be determined | Intervals of Increase | (-∞, -1.3) (3.2, 5) |
| Y-Intercepts | (0, 32) | Intervals of Decrease | (-1.3, 3.2) (5, ∞) |
| End Behavior | As x → -∞, W(x) → -∞<br>As x → ∞, W(x) → -∞ | | |