H Algebra II

Unit 3: Operations with Polynomials
Study Guide

Name__________________

What you need to know & be able to do	Things to remember	Problem	Problem
Naming, Adding, and Subtracting Polynomials	• Simplify polynomials and put in standard form before naming • The leading coefficient is the coefficient of the term with the highest degree	For each of the following, simplify, find the leading coefficient, and name by degree. 1. $x(x-3)+5x$ x^2+2x L.C. 1 quadratic 2. $2x^2-5x^4-3x^3+x^2$ $-5x^4-3x^3+3x^2$ L.C. -5 quartic	For each of the following, simplify, find the leading coefficient, and name by number of terms. 3. $(x^3+5x^2)-(x^2+4x)$ x^3+4x^2-4x L.C. 1 trinomial 4. $2x^2-5x^4-3x^2+x^2$ $-5x^4$ L.C. -5 monomial
Multiplying Polynomials	• Distribute all terms before combining like terms. • Use a concrete model (box) to help organize the multiplication	5. $(5x-3)(2x^2+4x-7)$ $10x^3+14x^2-47x+21$ 7. $(x+2)(3x-5)(2x-1)$ $6x^3-x^2-21x+10$	6. $-3(x-3)(x^4-5x^2+6)$ $-3x^5+9x^4+15x^3-45x^2-18x+54$ 8. $(3x^2y-2xy^2+y^3)(x^3-xy^2-8y^3)$ $3x^5y-2x^4y^2-2x^3y^3-22x^2y^4+15xy^5-8y^6$
Binomial Expansion	• Create Pascal's Triangle by adding the terms in the row above. • Pascal's Triangle determines the coefficients • The degree of the 1st term descends while the 2nd term ascends • For subtraction, consider the 2nd term negative.	Create Pascal's Triangle through the 8th row. 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1 1 6 15 20 15 6 1 1 7 21 35 35 21 7 1 1 8 28 56 70 56 28 8 1	9. $(x-y)^7$ $x^7-7x^6y+21x^5y^2-35x^4y^3+35x^3y^4-21x^2y^5+7xy^6-y^7$ 10. $(2x+y)^5$ $32x^5+80x^4y+80x^3y^2+40x^2y^3+10xy^4+y^5$ 11. $(5x-3y^2)^3$ $125x^3-225x^2y^2+135xy^4-27y^6$
Polynomial Long Divsion	• Make sure divisor and dividend are in standard form • Use 0's for placeholders for missing terms. • Put the remainder over the divisor	12. $(5x^3-6x^2+4x-3)\div(x-3)$ $5x^2+9x+31+\frac{90}{x-3}$	13. $(4x^3+8x^2-6)\div(2x+1)$ $2x^2+3x-1.5-\frac{4.5}{2x+1}$ or $2x^2+3x-\frac{3}{2}-\frac{9}{2(2x+1)}$

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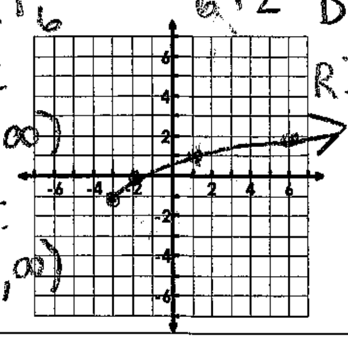
Factoring Polynomials	• Put the expression in STANDARD FORM • Remove any GCFS, including any negative leading coefficients. • Factor remaining polynomials using M-A chart, grouping, difference of squares, and sum/difference of cubes	14. $64x^4+343x$ $x(4x+7)(16x^2-28x+49)$	15. $25-400x^4$ $-25(2x+1)(2x-1)(4x^2+1)$
		16. x^4+8x^2-9 $(x^2+9)(x+1)(x-1)$	17. $2x^3-3x^2+4x-6$ $(x^2+2)(2x-3)$
		18. $-2x^6+24x^3-64$ $-2(x-2)(x^2+2x+4)(x^3-4)$	19. $x^6+7x^3y^3-8y^6$ $(x+2y)(x^2-2xy+4y^2)(x-y)(x^2+xy+y^2)$
Compositions of Functions	• Use the inside function to substitute into the outside function	Use $f(x)=2x-3$, $g(x)=x^2+3x$ 20. $f(g(-2))=-7$ 21. $g(f(5))=70$ 22. $g(g(3))=378$	Use $f(x)=2x-3$, $g(x)=x^2+3x$ 23. $f(g(x))=2x^2+6x-3$ 24. $g(f(x))=4x^2-6x$
Finding Inverses	• Switch x and y and solve for y. • Domain and Range switch	Find the inverse of the following. 25. $y=\frac{3-2x}{6}$ $f^{-1}(x)=-3x+\frac{3}{2}$ 26. $y=3(x-4)^3-12$ $f^{-1}(x)=4+\sqrt[3]{\frac{1}{3}(x+12)}$	Find the graph for the inverse of the following. Include restrictions. 27. $f(x)=(x+1)^2-3, x\geq-1$ x | f(x): -1 | -3; 0 | -2; 1 | 1; 2 | 6 x | $f^{-1}(x)$: -3 | -1; -2 | 0; 1 | 1; 6 | 2 D: $[-3,\infty)$ R: $[-1,\infty)$ D: $[-1,\infty)$ R: $[-3,\infty)$ 
Verifying Inverses	• Solve the compositions f(g(x)) = x and g(f(x)) = x	28. Verify the following are inverses. $y=\frac{4}{3}x+7$ $y=\frac{3}{4}x-5.25$ See paper	29. Verify the following are inverses. $y=3x^2+9$ $y=\sqrt{\frac{x}{3}-3}$

5.

	$2x^2$	$4x$	-7
$5x$	$10x^3$	$20x^2$	$-35x$
-3	$-6x^2$	$-12x$	21

$= \boxed{10x^3+14x^2-47x+21}$

6. $-3(x-3)(x^4-5x^2+6)$

$(-3x+9)(x^4-5x^2+6)$

	x^4	$-5x^2$	6
$-3x$	$-3x^5$	$15x^3$	$-18x$
9	$9x^4$	$-45x^2$	54

$= \boxed{-3x^5+9x^4+15x^3-45x^2-18x+54}$

7. $(x+2)(3x-5)(2x-1)$

	$2x$	-1
$3x$	$6x^2$	$-3x$
-5	$-10x$	5

$= (x+2)(6x^2-13x+5)$

	$6x^2$	$-13x$	5
x	$6x^3$	$-13x^2$	$5x$
2	$12x^2$	$-26x$	10

$= \boxed{6x^3-x^2-21x+10}$

8.

	$3x^2y$	$-2xy^2 +$	y^3
x^3	$3x^5y$	$-2x^4y^2$	x^3y^3
$-xy^2$	$-3x^3y^3$	$2x^2y^4$	$-xy^5$
$-8y^3$	$-24x^2y^4$	$16xy^5$	$-8y^6$

$= 3x^5y-2x^4y^2-2x^3y^3$
$-22x^2y^4+15xy^5-8y^6$

10. $(2x+y)^5 \Rightarrow$ 1 5 10 10 5 1

$1(2x)^5+5(2x)^4(y)+10(2x)^3(y)^2+10(2x)^2(y)^3+5(2x)(y)^4+1(y)^5$

$\boxed{32x^5+80x^4y+80x^3y^2+40x^2y^3+10xy^4+y^5}$

11. $(5x-3y^2)^3 \Rightarrow 1 \quad 3 \quad 3 \quad 1$

$1(5x)^3 + 3(5x)^2(-3y^2)^1 + 3(5x)^1(-3y^2)^2 + 1(-3y^2)^3$

$\boxed{125x^3 - 225x^2y^2 + 135xy^4 - 27y^6}$

12. $5x^2 + 9x + 31 + \frac{90}{x-3}$

$x-3 \,\overline{)\, 5x^3 - 6x^2 + 4x - 3}$

$-5x^3 + 15x^2$

$9x^2 + 4x$

$-9x^2 + 27x$

$31x - 3$

$-31x + 93$

90

13. $2x^2 + 3x - 1.5 - \frac{4.5}{2x+1}$

$2x+1 \,\overline{)\, 4x^3 + 8x^2 + 0x - 6}$

$-4x^3 - 2x^2$

$6x^2 + 0x$

$-6x^2 - 3x$

$-3x - 6$

$+3x + 1.5$

-4.5

14. $64x^4 + 343x$

$x(64x^3 + 343) \quad a = 4x \quad b = 7$

$x(4x+7)(16x^2 - 28x + 49)$

15. $25-400x^4$

$-400x^4+25$

$-25(16x^4-1)$

$-25(4x^2-1)(4x^2+1)$

$-25(2x+1)(2x-1)(4x^2+1)$

16. x^4+8x^2-9

$(x^4-1x^2)+(9x^2-9)$

$x^2(x^2-1)+9(x^2-1)$

$(x^2+9)(x^2-1)$

$(x^2+9)(x+1)(x-1)$

M	A
-9	8
+9, -1	

17. $2x^3-3x^2+4x-6$

$x^2(2x-3)+2(2x-3)$

$(x^2+2)(2x-3)$

18. $-2x^6+24x^3-64$

$-2(x^6-12x^3+32)$

$-2(x^6-4x^3-8x^3+32)$

$-2(x^3(x^3-4)-8(x^3-4))$

$-2(x^3-8)(x^3-4)$

$-2(x-2)(x^2+2x+4)(x^3-4)$

M	A
32	-12
-4	-8

19. $x^6+7x^3y^3-8y^6$

$(x^6-1x^3y^3)+(8x^3y^3-8y^6)$

$x^3(x^3-y^3)+8(x^3-y^3)$

$(x^3+8y^3)(x^3-y^3)$

$(x+2y)(x^2-2xy+4y^2)(x-y)(x^2+xy+y^2)$

M	A
-8	7
+8, -1	

20. $g(-2) = (-2)^2 + 3(-2) = -2$
$f(-2) = 2(-2) - 3 = \boxed{-7}$

21. $f(5) = 2(5) - 3 = 7$
$g(7) = (7)^2 + 3(7) = \boxed{70}$

22. $g(3) = (3)^2 + 3(3) = 18$
$g(18) = (18)^2 + 3(18) = \boxed{378}$

23. $f(g(x)) = 2(x^2 + 3x) - 3$
$= 2x^2 + 6x - 3$

24. $g(f(x)) = (2x-3)^2 + 3(2x-3)$
$(2x-3)(2x-3) + 6x - 9$
$4x^2 - 12x + 9 + 6x - 9$
$4x^2 - 6x$

25. $x = \frac{3-2y}{6}$
$6x = 3 - 2y$
$6x - 3 = -2y$
$y = -3x + \frac{3}{2}$

26. $x = 3(y-4)^3 - 12$
$x + 12 = 3(y-4)^3$
$\frac{1}{3}(x+12) = (y-4)^3$
$\sqrt[3]{\frac{1}{3}(x+12)} = y - 4$
$y = 4 + \sqrt[3]{\frac{1}{3}(x+12)}$

27. $f(x) = (x+1)^2 - 3$,
vertex: $(-1, -3)$
$x \geq -1$

x	$f(x)$
-1	-3
0	-2
1	1
2	6

D: $[-1, \infty)$
R: $[-3, \infty)$

$\xrightarrow[\text{x and y}]{\text{switch}}$

x	$f^{-1}(x)$
-3	-1
-2	0
1	1
6	2

28. $f(x) = \frac{4}{3}x + 7 \qquad g(x) = \frac{3}{4}x - 5.25$

$$f(g(x)) = \frac{4}{3}\left(\frac{3}{4}x - 5.25\right) + 7$$

$$= x - 7 + 7$$

$$= x$$

$$g(f(x)) = \frac{3}{4}\left(\frac{4}{3}x + 7\right) - 5.25$$

$$= x + 5.25 - 5.25$$

$$= x$$

$f(g(x)) = g(f(x)) = x$ so the 2 functions are inverses.

29. $f(x) = 3x^2 + 9$ $\quad g(x) = \sqrt{\frac{x}{3} - 3}$

$$f(g(x)) = 3\left(\sqrt{\frac{x}{3} - 3}\right)^2 + 9$$

$$= 3\left(\frac{x}{3} - 3\right) + 9$$

$$= x - 9 + 9$$

$$= x$$

$$g(f(x)) = \sqrt{\frac{(3x^2+9)}{3} - 3}$$

$$= \sqrt{x^2 + 3 - 3}$$

$$= \sqrt{x^2}$$

$$= \pm x$$

$g(f(x)) \neq x$, so the 2 functions are not inverses.