Unit 6 Review

1. $32^x - 5 = 3$
$32^x = 8$
$(2^5)^x = 2^3$
$5x = 3$
$\boxed{x = \frac{3}{5} = .6}$

2. $\log_5(15-2x) = \log_5(27-4x)$
$15 - 2x = 27 - 4x$
$15 + 2x = 27$
$2x = 12$
$\boxed{x = 6}$

3. $16 \ln x < 24$
$\ln x < 1.5$
$x < e^{1.5}$ $x > 0$
$\boxed{(0, e^{1.5})}$

4. $-5e^{-x} + 23 = 4$
$-5e^{-x} = -19$
$e^{-x} = 3.8$
$-x = \ln 3.8$
$\boxed{x = -\ln 3.8}$

5. $\log(x+7) + \log(x-4) = \log$
$\log(x+7)(x-4) = \log(2$
$\log(x^2+3x-28) = \log(2x$
$x^2 + 3x - 28 = 2x + 2$
$x^2 + x - 30 = 0$
$(x+6)(x-5) = 0$
$\boxed{x = \cancel{-6}, 5}$

6. $\frac{1}{2}\log_4 16x = 3$
$\log_4 16x = 6$
$16x = 4^6$
$16x = 4096$
$\boxed{x = 256}$

7. $4^{2x-1} = 5^{x+4}$
$\log_4 4^{2x-1} = \log_4 5^{x+4}$
$2x - 1 = (x+4)\log_4 5$
$2x - 1 = (\log_4 5)x + 4\log_4 5$
$-4\log_4 5 \quad -2x$
$-1 - 4\log_4 5 = x(\log_4 5 - 2)$
$\boxed{x = \frac{-1 - 4\log_4 5}{\log_4 5 - 2}}$

8. $-3\left(\frac{1}{6}\right)^{4x} + 18 \geq 3$
$-3\left(\frac{1}{6}\right)^{4x} \geq -15$
$\left(\frac{1}{6}\right)^{4x} \leq 5$
$4x \leq \log_{\frac{1}{6}} 5$
$x \leq \frac{1}{4}\log_{\frac{1}{6}} 5$
$\boxed{\left(-\infty, \frac{1}{4}\log_{\frac{1}{6}} 5\right]}$

9. $e^3 \approx 20.09$

$\boxed{3 \approx \ln 20.09}$

10. $\log_2 8 = 3$

$\boxed{8 = 2^3}$

11. $\ln 1.65 \approx \frac{1}{2}$

$\boxed{1.65 \approx e^{\frac{1}{2}}}$

12. $4^3 = 64$

$3 = \log_4 64$

13. $\log_5 \frac{1}{125} = x$

$\frac{1}{125} = 5^x$

$(5^{-3}) = 5^x$

$\boxed{-3}$

14. $\log .00001 = x$

$\log (10^{-5}) = x$

$-5 \log 10 = x$

$\boxed{-5}$

15. $\log_{\frac{1}{27}} 81 = x$

$81 = \frac{1}{27}^x$

$(3^4) = (3^{-3})^x$

$4 = -3x$

$\boxed{= -\frac{4}{3}}$

16. $9^{\log_3 6}$

$(3^2)^{\log_3 6}$

$3^{2 \log_3 6}$

$3^{\log_3 6^2}$

$6^2 = \boxed{36}$

17. $\log_{32} .25 = x$

$.25 = 32^x$

$\frac{1}{4} = 32^x$

$2^{-2} = 2^{5x}$

$-2 = 5x$

$\boxed{= \frac{-2}{5} \text{ or } -.4}$

18. $f(x) = \log_2 8^{x-3}$

D $= (x-3) \log_2 8$

$= (x-3) \cdot 3$

$= 3x - 9$

$\boxed{D: (-\infty, \infty) \ R: (-\infty, \infty)}$

$x = 3y - 9$

$x + 9 = 3y$

$y = \frac{1}{3}x + 3$

$\boxed{f^{-1}(x) = \frac{1}{3}x + 3}$

$\boxed{D: (-\infty, \infty) R: (-\infty, \infty)}$

19. $y = 5\left(\frac{1}{8}\right)^{x+9} - 7$

$\boxed{D: (-\infty, \infty) \quad R: (-7, \infty)}$

$x = 5\left(\frac{1}{8}\right)^{y+9} - 7$

$x + 7 = 5\left(\frac{1}{8}\right)^{y+9}$

$\frac{1}{5}(x+7) = \frac{1}{8}^{y+9}$

$\log_{\frac{1}{8}}\left(\frac{1}{5}(x+7)\right) = y + 9$

$\boxed{y = \log_{\frac{1}{8}}\left(\frac{1}{5}(x+7)\right) - 9 \quad D: (-7, \infty) \quad R: (-\infty, \infty)}$

20. $y = -e^{\frac{1}{7}x} + 16$

$D: \mathbb{R} \quad R: (-\infty, 16)$

$x = -e^{\frac{1}{7}y} + 16$

$x - 16 = -e^{\frac{1}{7}y}$

$-\frac{1}{7}y = \ln(-1(x-16))$

$\boxed{y = 7\ln(-(x-16)) \quad D: (-\infty, 16) \quad R(-\infty, \infty)}$

21. $f(x) = -\frac{1}{5}\ln(x-3) + 8$

$\boxed{D: (3, \infty) \quad R: (-\infty, \infty)}$

$x = -\frac{1}{5}\ln(y-3) + 8$

$x - 8 = -\frac{1}{5}\ln(y-3)$

$-5(x-8) = \ln(y-3)$

$\boxed{f^{-1}(x) = e^{-5(x-8)} + 3 \quad D: (-\infty, \infty) \quad R: (3, \infty)}$

22. $\log_4 6x^5y$

$\log_4 2 + \log_4 3 + \log_4 x^5 + \log_4 y$

$\log_4 (4)^{\frac{1}{2}} + \log_4 3 + 5\log_4 x + \log_4 y$

$\boxed{\frac{1}{2} + \log_4 3 + 5\log_4 x + \log_4 y}$

23. $\ln \frac{121x}{y^4 z}$

$\ln 11^2 + \ln x - \ln y^4 - \ln z$

$\boxed{2\ln 11 + \ln x - 4\ln y - \ln z}$

24. $\log_3 81a\sqrt{b}$

$\log_3 3^4 + \log_3 a + \log_3 b^{\frac{1}{2}}$

$4\log_3 3 + \log_3 a + \frac{1}{2}\log_3 b$

$\boxed{4 + \log_3 a + \frac{1}{2}\log_3 b}$

25. $\log 8 - \log 20 - 4\log x$

$\log 8 - \log 20 - \log x^4$

$\log\left(\frac{8}{20x^4}\right)$

$\boxed{\log\left(\frac{2}{5x^4}\right)}$

26. $\frac{1}{2}(\log_3 7 + \log_3 y)$

$\frac{1}{2}(\log_3 7y)$

$\log_3 (7y)^{\frac{1}{2}}$

$\boxed{\log_3 \sqrt{7y}}$

27. $\log_4 (x-1) - \frac{1}{5}\log_4 y - \log_4 m$

$\log_4 (x-1) - \log_4 y^{\frac{1}{5}} - \log_4 m$

$\log_4 \frac{(x-1)}{m\sqrt[5]{y}} \cdot \frac{\sqrt[5]{y^4}}{\sqrt[5]{y^4}}$

$\boxed{\log_4 \frac{(x-1)\sqrt[5]{y^4}}{my}}$

28. $\log_a (0.8) = \log_a \left(\frac{4}{5}\right)$

$\log_a 4 - \log_a 5$

$(1.8) - (2.4) = \boxed{-.6}$

29. $\log_a 20 = \log_a (4 \cdot 5)$

$\log_a 4 + \log_a 5$

$(1.8) + (2.4) = \boxed{4.2}$

30. $\log_a \sqrt{20} = \log_a (4 \cdot 5)^{\frac{1}{2}}$

$\frac{1}{2}(\log_a 4 + \log_a 5)$

$\frac{1}{2}(1.8 + 2.4) = \boxed{2.1}$

31. $\log_a 8 = \log_a (4)^{\frac{3}{2}}$

$\frac{3}{2}\log_a 4$

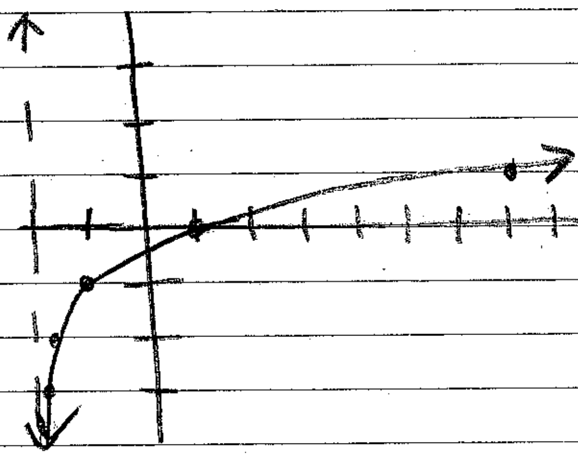
$\frac{3}{2}(1.8) = \boxed{2.7}$

32 $f(x) = \log_3 (x+2) - 1$ shift left 2 and down 1

parent: $\log_3 x$

x	$\log_3 x$	$x-2$	$y-1$
$\frac{1}{9}$	-2	$-\frac{17}{9}$	-3
$\frac{1}{3}$	-1	$-\frac{5}{3}$	-2
1	0	-1	-1
3	1	1	0
9	2	7	1

$x = -2$



$x = -2$

D: $(-2, \infty)$

R: $(-\infty, \infty)$

Asymp: $x = -2$

Int. of Inc: $(-2, \infty)$

X-int: $(1, 0)$

Y-int: $(0, \log_3 2 - 1)$

End: As $x \to -2$, $f(x) \to -\infty$

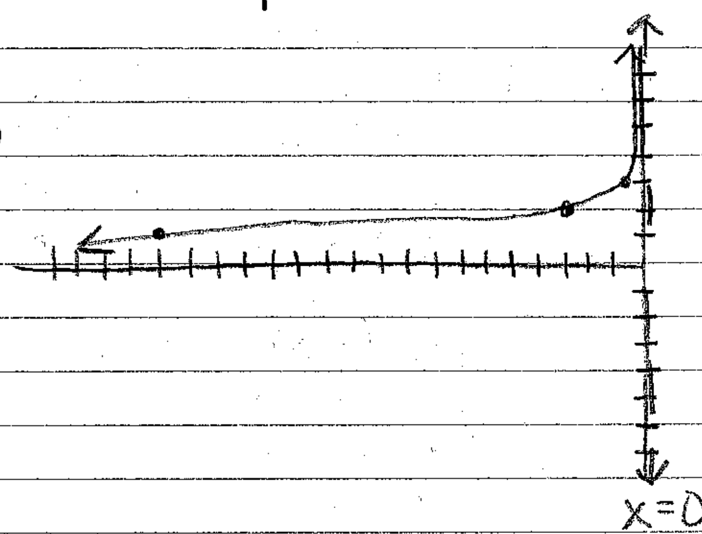
As $x \to \infty$, $f(x) \to \infty$

33. $y = \log_{\frac{1}{6}}(-2x) + 3$ reflect over y-axis, h. compression of $\frac{1}{2}$ shift up 3

parent: $\log_{\frac{1}{6}} x$

x	$\log_{\frac{1}{6}} x$	$-\frac{1}{2}x$	$y+3$
36	-2	-18	1
6	-1	-3	2
1	0	$-\frac{1}{2}$	3
$\frac{1}{6}$	1	$-\frac{1}{12}$	4
$\frac{1}{36}$	2	$-\frac{1}{72}$	5

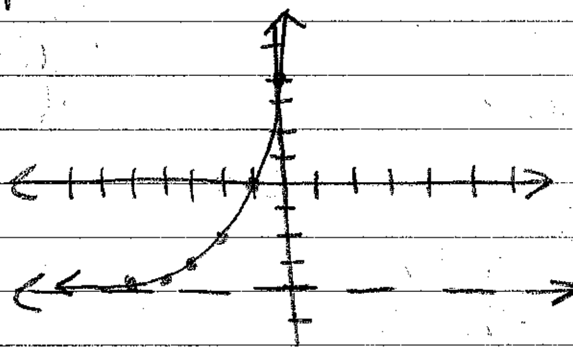
$y=0$



D: $(-\infty, 0)$
R: $(-\infty, \infty)$
Asymp: $x=0$
Int. of Inc: $(-\infty, 0)$
X-int: $(-108, 0)$
Y-int: none
End: As $x \to -\infty$, $y \to -\infty$
As $x \to 0$, $y \to \infty$

34. $f(x) = 2^{x+3} - 4$

x	2^x	$x-3$	$y-4$
-2	$.25$	-5	-3.75
-1	$.5$	-4	-3.5
0	1	-3	-3
1	2	-2	-2
2	4	-1	0

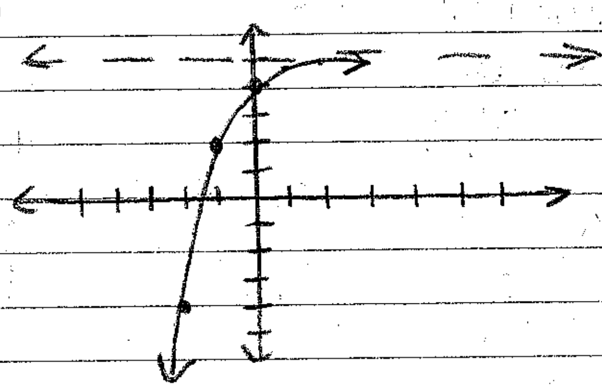


$y = -4$

D: $(-\infty, \infty)$
R: $(-4, \infty)$
Asymp: $y = -4$
X-int: $(-1, 0)$
Y-int: $(0, 4)$
End: As $x \to -\infty$, $f(x) \to -4$
As $x \to \infty$, $f(x) \to \infty$

35. $y = -\left(\frac{1}{3}\right)^x + 5$

x	$\frac{1}{3}^x$	x	$-y+5$
-2	9	-2	-4
-1	3	-1	2
0	1	0	4
1	$\frac{1}{3}$	1	$\frac{14}{3}$
2	$\frac{1}{9}$	2	$\frac{44}{9}$



$y = +5$

D: $(-\infty, \infty)$

R: $(-\infty, 5)$

Asymp: $y = 5$

X-int: $(-1.46, 0)$

Y-int: $(0, 4)$

End: As $x \to -\infty$, $y \to -\infty$

As $x \to \infty$, $y \to 5$

36. a. $5000(1+.056)^{10} = \boxed{\$8622.02}$

b. $5000\left(1+\frac{.056}{12}\right)^{12\cdot 10} = \boxed{\$8741.97}$

c. $5000e^{.056\cdot 10} = \boxed{\$8753.36}$

a. $3 = \left(1+\frac{.056}{4}\right)^{4t}$

$3 = (1.014)^{4t}$

$\log_{1.014} 3 = 4t$

$t = \log_{1.014} 3/4 = \boxed{19.76 \text{ years}}$

b. $3 = e^{.056t}$

$\ln 3 = .056t$

$t = \ln 3/.056 = \boxed{19.62 \text{ years}}$

$2 = e^{5r}$

$\ln 2 = 5r$

$r = \ln 2/5 = .1386 = \boxed{13.86\%}$

37. A. $y = 6.07(1+.018)^t$

$y = 6.07(1.018)^{20}$

$\boxed{y = 8.67 \text{ billion people}}$

B. $8 = 6.07(1.018)^t$

$\frac{8}{6.07} = 1.018^t$

$\log_{1.018}(1.318) = t$

$t = 15.48$ years

$\boxed{\text{During the year 2015}}$

C. $1 = 6.07(1.018)^t$

$\frac{1}{6.07} = 1.018^t$

$\log_{1.018}(.165) = t$

$t = -101.09$

$2000 - 101.09 = 1898.91$

$\boxed{\text{At the end of 1898}}$

38. $44129 = 27216e^{20k}$

$\frac{44129}{27216} = e^{20k}$

$\ln\left(\frac{44129}{27216}\right) = 20k$

$k = \frac{1}{20}\ln\left(\frac{44129}{27216}\right) \approx .024$ (store value)

$P(40) = 27216e^{.024(40)} \approx 71552.35$

$\boxed{\text{In 2010, there were about 71,600 people in Marietta}}$