Honors Algebra II
Unit 4A Polynomials
Review for Test

Name________________________

Date____________Block____

I. Determine if the functions are even, odd or neither. <u>Use algebra to prove how you come to your conclusion.</u> (No Calculator)

a. $f(x) = x^3 - 3x^2 + 2x - 1$

$f(-x) = (-x)^3 - 3(-x)^2 + 2(-x) - 1$
$f(-x) = -x^3 - 3x^2 - 2x - 1$
$f(-x) \neq f(x) and\ f(-x) \neq -f(x),$
$so\ f(x)\ has\ neither\ even\ nor\ odd\ symmetry.$

b. $g(x) = 2x^4 - x^2 + 2$

$g(-x) = 2(-x)^4 - (-x)^2 + 2$
$g(-x) = 2x^4 - x^2 + 2$
$g(-x) = g(x),\ so\ g(x)\ has\ even\ symmetry$

c. $h(x) = 4x^4 + 2x^2 + x$

$h(-x) = 4(-x)^4 + 2(-x)^2 + (-x)$
$h(-x) = 4x^4 + 2x^2 - x$
$h(-x) \neq h(x) and\ h(-x) \neq -h(x),$
$so\ h(x)\ has\ neither\ even\ nor\ odd\ symmetry$

d. $f(x) = 5x^5 + 3x^3 + x$

$f(-x) = 5(-x)^5 + 3(-x)^3 + (-x)$
$f(-x) = -5x^5 - 3x^3 - x$
$f(-x) = -f(x),\ so\ g(x)\ has\ odd\ symmetry$

2. For the functions given, identify: Domain; Range; Intercepts; Relative maximum and/or minimum; Absolute maximum and/or minimum; Intervals of increase and decrease; End behavior. (Calculator)

a. $f(x) = \frac{1}{4}x^4 - 2x^2 + 16$

Domain: (-∞, ∞)
Range: [12.73, ∞)
X-Intercepts: none
Y-Intercepts: (0, 16)
Max: (0, 16) (relative)
Min: (-2, 12) (2, 12) (absolute)
Inc: (-2, 0) (2, ∞)
Dec: (-∞, -2) (0, 2)
End Behavior: As x → -∞, f(x) →∞
As x → ∞, f(x) →∞

b. $f(x) = -2x^3 + 6x^2 - \frac{9}{2}x$

Domain: (-∞, ∞)
Range: (-∞, ∞)
X-Intercepts: (0, 0) (1.5, 0)
Y-Intercepts: (0, 0)
Max: (1.5, 0) (relative)
Min: (.5, -1) (relative)
Inc: (.5, 1.5)
Dec: (-∞, .5) (1.5, ∞)
End Behavior: As x → -∞, f(x) →∞
As x → ∞, f(x) →-∞

3. For the functions given, find the leading coefficient, degree of the polynomial function, list the intercepts and determine any multiplicity. Sketch a graph. (No Calculator)

a. $f(x) = -2(x-2)(x+3)^3(x+1)^2$

Leading Coefficient: -2
Degree: 6
X-Intercepts: (2, 0), (-3, 0) mult. 3, (-1, 0) mult. 2
Y-Intercepts: $-2(-2)(3)^3(1)^2$ = (0, 108)

b. $g(x) = 3x(x+2)^4(x+1)^3(x-8)$

Leading Coefficient: 3
Degree: 9
X-Int: (0, 0), (-2, 0) mult. 4, (-1, 0) mult.3, (8, 0)
Y-Int: (0, 0)

4. Find the standard form of the new polynomial with the given transformation(s). Describe the transformation(s). (No Calculator) ***$P(x) = 2x^3 - 5x + 1$***

a. T(x) = -2P(x)
   Reflected over x-axis, vertically stretched by 2
   $T(x) = -2(2x^3 - 5x + 1)$
   $T(x) = -4x^3 + 10x - 2$

b. T(x) = P(x – 4) + 1
   Shifted right 4, shifted up 1
   $T(x) = [2(x-4)^3 - 5(x-4) + 1] + 1$
   $T(x) = [2(x^3 - 12x^2 + 48x - 64) - 5x + 20 + 1] + 1$
   $T(x) = [2x^3 - 24x^2 + 96x - 128 - 5x + 21] + 1$
   $T(x) = [2x^3 - 24x^2 + 91x - 107] + 1$
   $T(x) = 2x^3 - 24x^2 + 91x - 106$

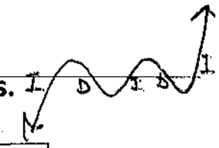
c. T(x) = -P(x + 2) + 3
   Shifted left 2, reflected over x-axis, shifted up 3
   $T(x) = -(2(x+2)^3 - 5(x+2) + 1) + 3$
   $T(x) = -(2(x^3 + 6x^2 + 12x + 8) - 5x - 10 + 1) + 3$
   $T(x) = -(2x^3 + 12x^2 + 24x + 16 - 5x - 9) + 3$
   $T(x) = -(2x^3 + 12x^2 + 19x + 7) + 3$
   $T(x) = -2x^3 - 12x^2 - 19x - 4$

d. T(x) is the transformation of P(x) that has been moved 3 right and vertically shrunk by ½
   T(x) = 1/2P(x – 3)
   $T(x) = 1/2(2(x-3)^3 - 5(x-3) + 1)$
   $T(x) = 1/2(2(x^3 - 9x^2 + 27x - 27) - 5x + 16)$
   $T(x) = 1/2(2x^3 - 18x^2 + 49x - 38)$
   $T(x) = x^3 - 9x^2 + 24.5x - 19$

5. Complete the table below. Describe the characteristics for each of the following transformations. (No Calculator)

Leading Coefficient	4	Absolute Maximum	None
Degree	5	Relative Maximum	(-2, 648)(2.6, 56.2)
Domain	(-∞, ∞)	Absolute Minimum	None
Range	(-∞, ∞)	Relative Minimum	(1,0) (4,0)
X-Intercepts	(-3, 0) (1, 0)mult.2 (4,0)mult.2	Intervals of Increase	(-∞, -2)(1, 2.6)(4, ∞)
Y-Intercepts	(0, 96)	Intervals of Decrease	(-2, 1)(2.6, 4)
End Behavior	As x → -∞, P(x) → -∞ As x → ∞, P(x) → ∞		

5. Complete the table below. Describe the characteristics for each of the following transformations. (No Calculator)



Leading Coefficient	4	Absolute Maximum	NONE
Degree	5	Relative Maximum	(-2, 648)(2.6, 56.2)
Domain	$(-\infty, \infty)$	Absolute Minimum	NONE
Range	$(-\infty, \infty)$	Relative Minimum	(1,0) (4,0)
X-Intercepts	(-3, 0) (1, 0)mult.2 (4,0)mult.2	Intervals of Increase	$(-\infty,-2)\cup(1,2.6)\cup(4,\infty)$
Y-Intercepts	(0, 96)	Intervals of Decrease	$(-2,1)\cup(2.6,4)$
End Behavior	As $x \to -\infty$, $P(x) \to -\infty$ As $x \to \infty$, $P(x) \to \infty$		

a. $T(x) = -P(x)$ Reflect over x-axis (Mult. y by -1)

Leading Coefficient	-4	Absolute Maximum	NONE
Degree	SAME	Relative Maximum	(1,0) (4,0)
Domain	SAME	Absolute Minimum	NONE
Range	SAME	Relative Minimum	(-2,-648) (2.6,-56.2)
X-Intercepts	SAME	Intervals of Increase	$(-2,1)\cup(2.6,4)$
Y-Intercepts	(0,-96)	Intervals of Decrease	$(-\infty,-2)\cup(1,2.6)$ $\cup(4,\infty)$
End Behavior	As $x \to -\infty$, $T(x) \to \infty$ As $x \to \infty$, $T(x) \to -\infty$		

b. $T(x) = P(x-3)$ Shift right 3 (Add 3 to x)

Leading Coefficient	SAME	Absolute Maximum	NONE
Degree	SAME	Relative Maximum	(1,648) (5.6,56.2)
Domain	SAME	Absolute Minimum	NONE
Range	SAME	Relative Minimum	(4,0) (7,0)
X-Intercepts	(0,0) (4,0)m.2 (7,0)m.2	Intervals of Increase	$(-\infty,1)\cup(4,5.6)\cup(7,\infty)$
Y-Intercepts	CBD (happens to be (0,0))	Intervals of Decrease	$(1,4)\cup(5.6,7)$
End Behavior	SAME		

c. $T(x) = P(x) - 4$ Shift down 4 (subt. 4 from y)

Leading Coefficient	SAME	Absolute Maximum	NONE
Degree	SAME	Relative Maximum	(-2,644) (2.6,52.2)
Domain	SAME	Absolute Minimum	NONE
Range	SAME	Relative Minimum	(1,-4) (4,-4)
X-Intercepts	CBD	Intervals of Increase	SAME
Y-Intercepts	(0,92)	Intervals of Decrease	SAME
End Behavior	SAME		

d. $T(x) = -3P(x) + 2$ Reflect over x-axis V. Stretch by 3, shift up 2 (Mult. y by -3 and add 2)

Leading Coefficient	−12	Absolute Maximum	NONE
Degree	SAME	Relative Maximum	(1,2) (4,2)
Domain	SAME	Absolute Minimum	NONE
Range	SAME	Relative Minimum	(-2,-1942)(2.6,-166.6)
X-Intercepts	CBD	Intervals of Increase	(-2,1)(2.6,4)
Y-Intercepts	(0,-286)	Intervals of Decrease	(-∞,-2)(1,2.6) (4,∞)
End Behavior	As $x \to -\infty$, $T(x) \to \infty$ As $x \to \infty$, $T(x) \to -\infty$		

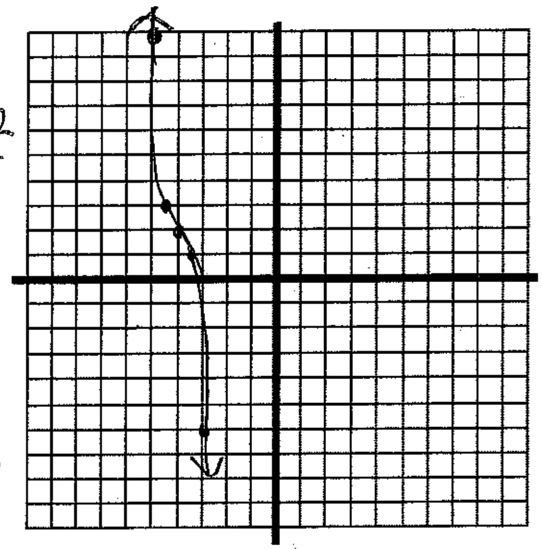
6. Describe the transformations, create a table, and sketch a graph of transformations of cubic and quartic parent functions. (No calculator)

a. $f(x) = -(2x+8)^3 + 2 = -(2(x+4))^3 + 2$

- reflect over x-axis (y)
- h. comp. by 1/2 (x)
- shift left 4 (x)
- shift up 2

parent

x	x^3	$\frac{1}{2}x-4$	$-y+2$
-2	-8	-5	10
-1	-1	-4.5	3
0	0	-4	2
1	1	-3.5	1
2	8	-3	-6

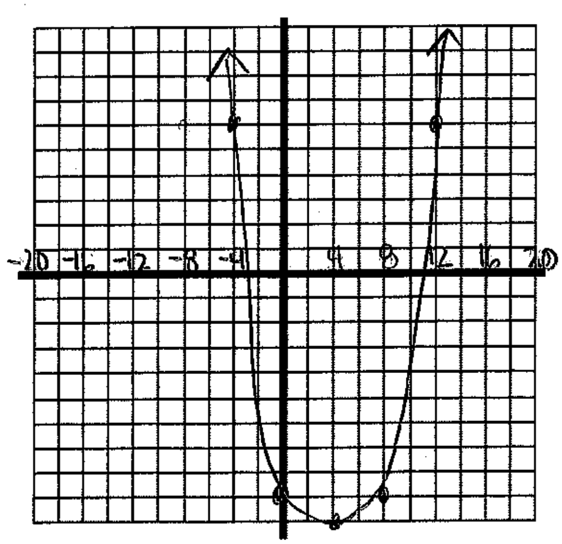


b. $h(x) = \left(\frac{1}{4}x - 1\right)^4 - 10 = \left(\frac{1}{4}(x-4)\right)^4 - 10$

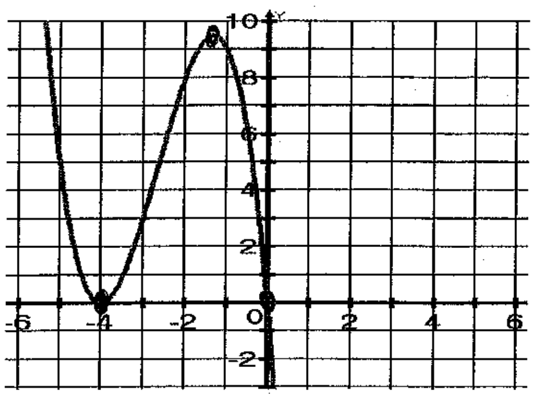
- h. stretch by 4 (x)
- shift right 4 (x)
- shift down 10

parent

x	x^4	$4x+4$	$y-10$
-2	16	-4	6
-1	1	0	-9
0	0	4	-10
1	1	8	-9
2	16	12	6



7. Describe the transformations, create a table, and sketch a graph for the transformation of a given graphical function. (Calculator)



$f(x)$

(-4, 0)

(-1.5, 9.5)

(0, 0)

a. $G(x) = -\frac{1}{2}f(x)$

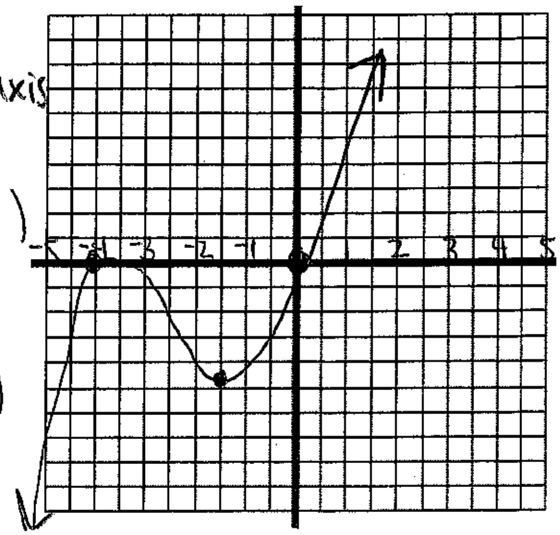
- reflect over x-axis
- v. comp. by 1/2

$g(x) = (x, -\frac{1}{2}y)$

(-4, 0)

(-1.5, -4.75)

(0, 0)



b. $H(x) = f(x-2) - 4$

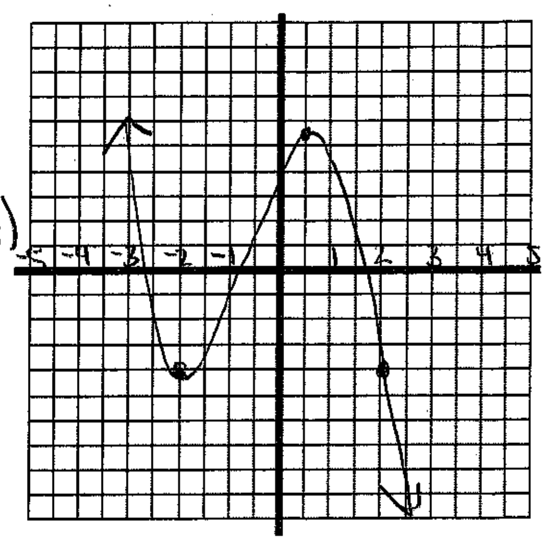
- shift right 2
- shift down 4

$H(x) = (x+2, y-4)$

(-2, -4)

(0.5, 5.5)

(2, -4)



$\frac{1}{2}f\left(\frac{1}{3}(x-6)\right)+2$

c. $K(x) = \frac{1}{2}f\left(\frac{1}{3}x - 2\right) + 2$

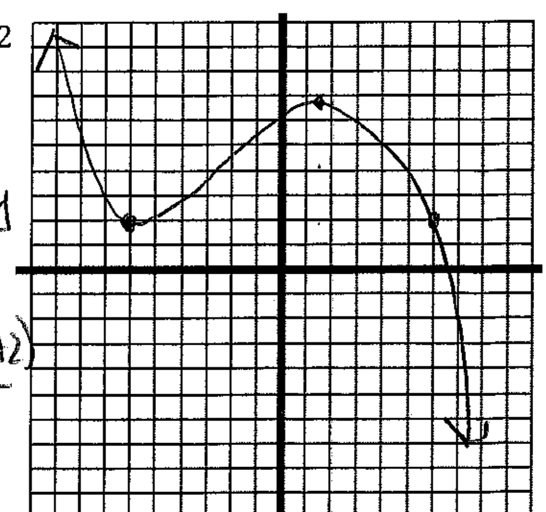
- v. comp. by 1/2
- h. stretch by 3
- shift right 6 and up 2

$K(x) = (3x+6, \frac{1}{2}y+2)$

(-6, 2)

(1.5, 6.75)

(6, 2)

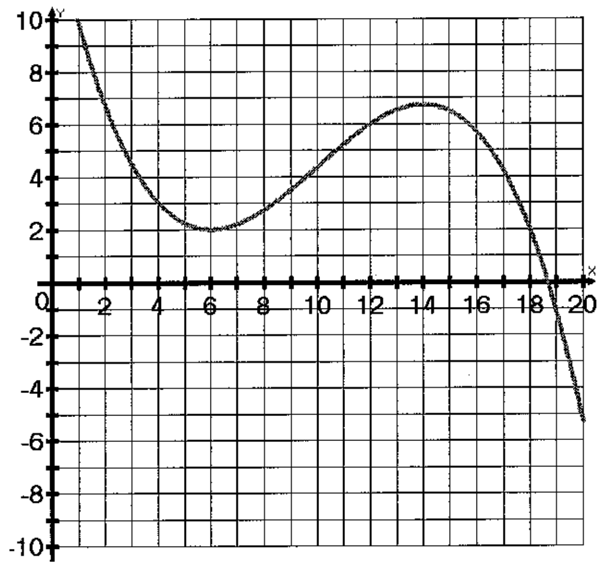


c. $K(x) = \frac{1}{2} f\left(\frac{1}{3}x - 2\right) + 2$

$$K(x) = \frac{1}{2} f\left(\frac{1}{3}(x-6)\right) + 2$$

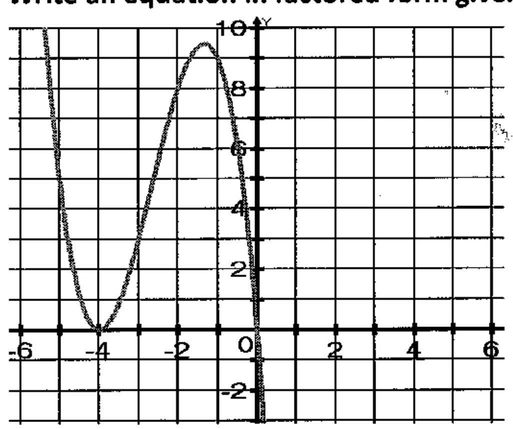
shift right 6, *horizontally stretch by* 3, *vertically compress by* $\frac{1}{2}$, *shift up* 2

F(x)	G(x)
(-4, 0)	(6, 2)
(-1.5, 9.5)	(13.5, 6.75)
(0, 0)	(18, 2)



8. Write an equation in factored form given a polynomial graph. Expand that equation to standard form.

a.

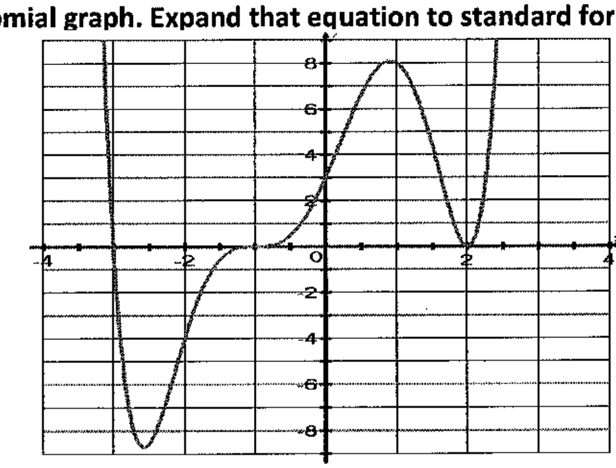


$f(x) = -x(x+4)^2$

$f(x) = -x(x^2 + 8x + 16)$

$f(x) = -x^3 - 8x^2 - 16x$

b.



$g(x) = \frac{1}{4}(x+3)(x+1)^3(x-2)^2$

$g(x) = \frac{1}{4}(x+3)(x^3 + 3x^2 + 3x + 1)(x^2 - 4x + 4)$

$g(x) = \frac{1}{4}(x^3 - x^2 - 8x + 12)(x^3 + 3x^2 + 3x + 1)$

$g(x) = \frac{1}{4}(x^6 + 2x^5 - 8x^4 - 14x^3 + 11x^2 + 28x + 12)$

$g(x) = \frac{1}{4}x^6 + \frac{1}{2}x^5 - 2x^4 - \frac{7}{2}x^3 + \frac{11}{4}x^2 + 7x + 3$

9. Solve the following expressions systems of equations graphically. (Calculator)

a. $f(x) = 15x^2 - x - 5$
$g(x) = -3x^3$

(-.58, .58) (.58, -.58)

b. $f(x) = 2x^4 - 12x^2 + 8x^3$
$g(x) = 48x$

(-4, -192) (-2.45, -117.58) (0, 0) (2.45, 117.58)

c. $f(x) = 5x^4 - 18x^2 + 36$
$g(x) = 4x^4 - 5x^2$

(-3, 279) (-2, 44) (2, 44) (3, 279)

d. $f(x) = 18x^4 - 5$
$g(x) = -27x^2$

(-.41, -4.5) (.41, -4.5)

10. Applications. (Calculator)

a. *A hot-air balloon rises at an increasing rate as its altitude increases. Its altitude, in feet, can be modeled by the function y = 0.025t² + 2t, where t is time in seconds. How long will it take for the balloon to reach an altitude of 800 feet?*

$$800 = 0.025t^2 + 2t$$
$$y = 0.025t^2 + 2t$$
$$y = 800$$
$$143.3\ Seconds$$ (intersection of the two)

b. *The US Postal Service requires that the sum of the height and the girth of a parcel is no greater than 108 inches. The girth is the distance around the package, 2l + 2w. Suppose that a box has the same length and width. The volume of the box is given by V(x) = x(x)(108 – 4x), where x is the length.*

1) Write the volume of the box as a polynomial function in standard form.
$$V(x) = -4x^3 + 108x^2$$
2) Find the maximum possible volume for such a box.
11,664 in^3
The y-value at the relative maximum in the 1st quadrant
3) Find the volume of the box if its length is 24 inches.
$$V(24) = -4(24)^3 + 108(24)^2 = 6912in^3$$
4) Write the volume function for an acceptable box whose length is twice its width.
$$2w = l$$
$$l = x$$
$$w = \frac{x}{2}$$
$$girth = 2x + 2\left(\frac{x}{2}\right)$$
$$girth = 2x + x = 3x$$
$$height = 108 - 3x$$
$$V(x) = x\left(\frac{x}{2}\right)(108 - 3x)$$
$$V(x) = -\frac{3}{2}x^3 + 54x^2$$