Honors Algebra II

Name ____________________

Unit 5A Review – Rational Functions

1. Simplifying Rationals
   a. $\frac{5x^3-30x^2-35x}{x^3+x^2-4x-4}$

2. Multiplying and Dividing Rationals
   a. $\frac{x^2-5x}{x^2+5x+4} \cdot \frac{6x^2+12x+6}{9x^3-45x^2}$
   b. $\frac{36xy^3}{27x^4} \div \frac{12xy}{20y^2}$

3. Adding and Subtracting Rationals
   a. $\frac{x}{2x-2} + \frac{x-5}{x^2-3x-4}$
   b. $\frac{5}{6x^2-12x} - \frac{2}{3x}$

4. Solving Rationals (check for extraneous solutions)
   a. $\frac{1}{r-5} = \frac{7}{2r}$
   b. $\frac{4}{x} + 6 = \frac{1}{x^2}$
   c. $\frac{x-4}{x+2} + \frac{2}{x-2} = \frac{17}{x^2-4}$
   d. $\frac{1}{1+c} - \frac{1}{2+c} = \frac{1}{4}$

5. Graphing Rationals with Transformations. Find all characteristics without a calculator. (Domain, range, max, min, intercepts, intervals of increase/decrease/constant, end behavior)
   a. $\frac{1}{x-3} + 2$
   b. $\frac{-1}{2x}$
   c. $\frac{4}{2-x} - 1$

6. Graphing and Characteristics of other Rationals (Domain, Asymptotes, Holes, Discontinuities, x/y-intercepts without calculator).
   a. $\frac{5x^2-10x}{2x^3-10x^2+12x}$
   b. $\frac{10x^2-40}{2x^2-x-6}$
   c. $\frac{x^3+x^2-9x-9}{x^2-2x-3}$

7. Solving Inequalities. Plot on number line and write answer in interval notation.
   a. $\frac{4}{x-3} > 6$
   b. $\frac{x+6}{x-4} \leq -3$
   c. $\frac{x+1}{x-1} + \frac{2}{x} \geq 1$
   d. $\frac{x-3}{3x} \geq \frac{1}{3x^2+9x} + \frac{1}{x+3}$

8. Verifying Inverses

   a. Verify that $f(x) = \frac{3}{x-2} + 4$ and $g(x) = \frac{3}{x-4} + 2$ are inverses.

9. Find the inverse of the following functions. Also find the domain and range of both the original function and the inverse.

   a. $g(x) = \frac{5}{x-4} + 1$

   b. $h(x) = \frac{3x-2}{4x+1}$

1a. $\dfrac{5x^3-30x^2-35x}{(x^3+x^2)(-4x-4)}$

$\dfrac{5x(x^2-6x-7)}{(x^2-4)(x+1)}$

$\dfrac{5x(x-7)(x+1)}{(x-2)(x+2)(x+1)}$

$\boxed{\dfrac{5x(x-7)}{(x-2)(x+2)}}$

2a. $\dfrac{x^2-5x}{x^2+5x+4} \cdot \dfrac{6x^2+12x+6}{9x^3-45x^2}$

$\dfrac{x\cancel{(x-5)} \cdot {}^{2}\cancel{6}\cancel{(x+1)}(x+1)}{(x+4)\cancel{(x+1)} \cdot {}_{3}\cancel{9}x^{\cancel{2}}\cancel{(x-5)}}$

$\boxed{\dfrac{2(x+1)}{3x(x+4)}}$

2b. $\dfrac{36xy^3}{27x^4} \div \dfrac{12xy}{20y^2}$

$\dfrac{36xy^3}{27x^4} \cdot \dfrac{20y^2}{12xy}$

$\dfrac{\cancel{2}\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{3}(2\cdot 2\cdot 5)\ x\cdot y^5}{\cancel{3}\cdot\cancel{3}\,(3)\,\cancel{2}\cdot\cancel{2}\,(3)\ x^5\cdot y}$

$\boxed{\dfrac{20y^4}{9x^4}}$

3a. $\dfrac{x}{2(x-1)} + \dfrac{x-5}{(x-4)(x+1)}$

$\dfrac{x(x-4)(x+1)+2(x-1)(x-5)}{2(x-1)(x-4)(x+1)}$

$\dfrac{x^3-3x^2-4x+2x^2-12x+10}{2(x-1)(x-4)(x+1)} = \boxed{\dfrac{x^3-x^2-16x+10}{2(x-1)(x-4)(x+1)}}$

3b $\frac{5}{6x^2-12x} - \frac{2}{3x}$

$\frac{5}{6x(x-2)} - \frac{2}{3x} \cdot \frac{2(x-2)}{2(x-2)}$

$\frac{5-4x+8}{6x(x-2)}$

$\boxed{\frac{-4x+13}{6x(x-2)}}$

4a. $\frac{1}{r-5} = \frac{7}{2r}$

$2r = 7r - 35$

$-5r = -35$

$\boxed{r=7}$

4b $\left(\frac{4}{x} + 6 = \frac{1}{x^2}\right) \cdot x^2$

$4x + 6x^2 = 1$

$6x^2 + 4x - 1 = 0$

$x = \frac{-4 \pm \sqrt{16+24}}{12}$

$x = \frac{-4 \pm \sqrt{40}}{12}$

$x = \frac{-4 \pm 2\sqrt{10}}{12}$

$\boxed{x = \frac{-2 \pm \sqrt{10}}{6}}$

4c $\frac{x-4}{x+2} + \frac{2}{x-2} = \frac{17}{(x-2)(x+2)}$

$(x-4)(x-2) + 2(x+2) = 17$

$x^2 - 6x + 8 + 2x + 4 = 17$

$x^2 - 4x + 12 = 17$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$\boxed{x = -1, 5}$

4d $\frac{1}{1+c} - \frac{1}{2+c} = \frac{1}{4}$

$4(2+c) - 4(1+c) = (2+c)(1+c)$

$8 + 4c - 4 - 4c = c^2 + 3c + 2$

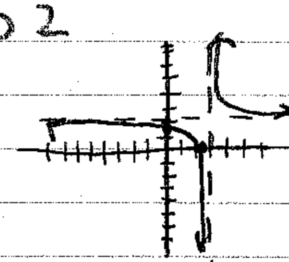
$4 = c^2 + 3c + 2$

$0 = c^2 + 3c - 2$

$c = \frac{-3 \pm \sqrt{9+8}}{2}$

$\boxed{c = \frac{-3 \pm \sqrt{17}}{2}}$

5a $\frac{1}{x-3} + 2$ shifted right 3, shifted up 2



H.A. $y = 2$ V.A. $x = 3$

D: $\{x \mid x \neq 3\}$

R: $\{y \mid y \neq 2\}$

max/min: none

x-int: $0 = \frac{1}{x-3} + 2$

$-2 = \frac{1}{x-3}$

$-2x + 6 = 1$

$-2x = -5$

$x = \frac{5}{2}$

$(\frac{5}{2}, 0)$

y-int: $\frac{1}{0-3} + 2$

$(0, \frac{5}{3})$

Inc: none

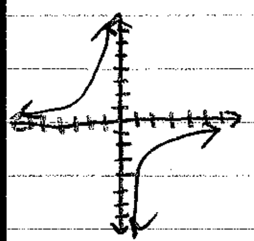
Dec: $(-\infty, 3) \cup (3, \infty)$

Const: none

End Behavior: As $x \to -\infty$, $y \to 2$

As $x \to \infty$, $y \to 2$

5b. $-\frac{1}{2x}$ reflect over x-axis, shrink by $\frac{1}{2}$



V.A. $x=0$ H.A. $y=0$

D: $\{x \mid x \neq 0\}$

R: $\{y \mid y \neq 0\}$

max/min: none

x-int: none

y-int: none

Inc: $(-\infty, 0) \cup (0, \infty)$

Dec: none

Const: none

End Behavior: As $x \to -\infty$, $y \to 0$

As $x \to \infty$, $y \to 0$

5c. $\frac{4}{2-x} - 1 = \frac{4}{-1(x-2)} - 1$

shift right 2, reflect over x-axis, stretch by 4, shift down 1

V.A. $x=2$ H.A. $y=-1$

D: $\{x \mid x \neq 2\}$

R: $\{y \mid y \neq -1\}$

max/min: none

x-int: $0 = \frac{4}{-1(x-2)} - 1$

$1 = \frac{4}{-1(x-2)}$

$-1 = \frac{4}{x-2}$

$-x+2 = 4$

$-x = 2$

$x = -2$

$(-2, 0)$

y-int: $\frac{4}{2-0} - 1$

$(0, 1)$

Inc: $(-\infty, 2) \cup (2, \infty)$

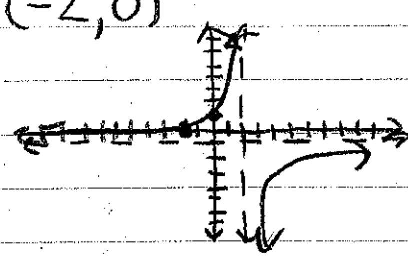
Dec: none

Const: none

End Behavior:

As $x \to -\infty$, $y \to -1$

As $x \to \infty$, $y \to -1$



6a. $\frac{5x^2-10x}{2x^3-10x^2+12x} = \frac{5\cancel{x}\cancel{(x-2)}}{2\cancel{x}\cancel{(x-2)}(x-3)} = \frac{5}{2(x-3)}$

V.A. $x=3$

H.A. $y=0$

Holes: $(2, -\frac{5}{2})$ $(0, -\frac{5}{6})$

Discontinuous at $x = 0, 2,$ and 3

X-int: none (asymptote)

y-int: none (hole)

6b. $\frac{10x^2-40}{2x^2-x-6} = \frac{10\cancel{(x-2)}(x+2)}{(2x+3)\cancel{(x-2)}} = \frac{10(x+2)}{2x+3}$

V.A. $x = -\frac{3}{2}$

H.A. $y=5$

Holes: $(2, \frac{40}{7})$

Discontinuous at $x = -\frac{3}{2}$ and 2

X-int: $(-2, 0)$

Y-int: $(0, \frac{20}{3})$

6c. $\frac{x^3+x^2-9x-9}{x^2-2x-3} = \frac{(x+3)\cancel{(x-3)}\cancel{(x+1)}}{\cancel{(x-3)}\cancel{(x+1)}} = x+3$

no asymptotes

Holes: $(-1, 2)$ $(3, 6)$

Discontinuous at $x = -1$ and 3

X-int: $(-3, 0)$

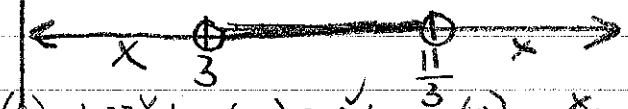
Y-int: $(0, 3)$

7a. $\frac{4}{x-3} > 6 \quad x \neq 3$

$4 = 6x - 18$

$22 = 6x$

$x = \frac{11}{3}$



$\boxed{(3, \frac{11}{3})}$

(0) $-1.33 \not> 6$ (3.2) $20 > 6$ ✓ (4) $4 \not> 6$

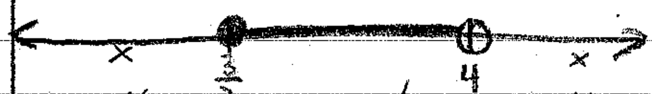
Use calculator!

7b. $\frac{x+6}{x-4} \leq -3 \quad x \neq 4$

$x + 6 = -3x + 12$

$4x = 6$

$x = \frac{3}{2}$



$\boxed{[\frac{3}{2}, 4)}$

(0) $-1.5 \not\leq -3$ (3) $-9 \leq -3$ ✓ (5) $11 \not\leq -3$

7c. $\left(\frac{x+1}{x-1} + \frac{2}{x} \geq 1\right)(x(x-1))$

$x \neq 0, 1$

$x^2 + x + 2x - 2 \geq x^2 - x$

$\cancel{x^2} + 3x - 2 = \cancel{x^2} - x$

$4x - 2 = 0$

$4x = 2$

$x = \frac{1}{2}$

$\boxed{(0, \frac{1}{2}] \cup (1, \infty)}$

(-1) $-2 \geq 1$ (.25) $6.33 \geq 1$ (.75) $-4.33 \geq 1$ (1.5) $6.33 \geq 1$

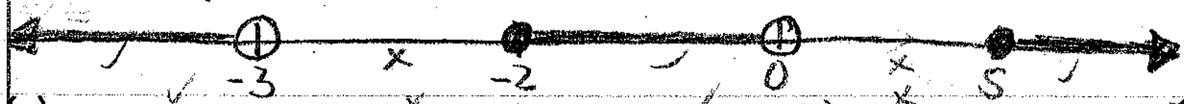
7d. $\left(\frac{x-3}{3x} \ge \frac{1}{3x(x+3)} + \frac{1}{x+3}\right) \cdot (3x(x+3)) \quad x \ne 0, -3$

$x^2-9 = 1+3x$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=-2, 5$



$(-4)\ .583 \ge .92 \quad (-2.5)\ .73 \ge 1.73 \quad (-1)\ 1.33 \ge .33 \quad (2)\ -.17 \ge .23 \quad (6)\ .17 \ge .12$

$\boxed{(-\infty,-3) \cup [-2,0) \cup [5,\infty)}$

8a. $f(x) = \frac{3}{x-2} + 4 \qquad g(x) = \frac{3}{x-4} + 2$

$f(g(x)) = \frac{3}{\left(\frac{3}{x-4}+2\right)-2} + 4 = \frac{3}{\frac{3}{x-4}} + 4 = 3 \cdot \frac{x-4}{3} + 4$

$= x-4+4 = x$

$g(f(x)) = \frac{3}{\left(\frac{3}{x-2}+4\right)-4} + 2 = \frac{3}{\frac{3}{x-2}} + 2 = 3 \cdot \frac{x-2}{3} + 2$

$= x-2+2 = x$

$f(g(x)) = g(f(x)) = x$, so $f(x)$ and $g(x)$ are inverses.

9a. $g(x) = \frac{5}{x-4} + 1$ D: $\{x \mid x \neq 4\}$ R: $\{y \mid y \neq 1\}$

$x = \frac{5}{y-4} + 1$

$x - 1 = \frac{5}{y-4}$

$y - 4 = \frac{5}{x-1}$

$$\boxed{g^{-1}(x) = \frac{5}{x-1} + 4 \quad D: \{x \mid x \neq 1\} \quad R: \{y \mid y \neq 4\}}$$

9b. $h(x) = \frac{3x-2}{4x+1}$ D: $\{x \mid x \neq -\frac{1}{4}\}$ R: $\{y \mid y \neq \frac{3}{4}\}$

$\left(x = \frac{3y-2}{4y+1}\right)(4y+1)$

$4xy + x = 3y - 2$

$4xy - 3y = -x - 2$

$y(4x - 3) = -x - 2$

$$\boxed{h^{-1}(x) = \frac{-x-2}{4x-3} \quad D: \{x \mid x \neq \frac{3}{4}\} \quad R: \{y \mid y \neq -\frac{1}{4}\}}$$