Name __AK__

A. Find both $f(g(x))$ and $g(f(x))$.

1. $f(x)=x^2-1;\ g(x)=\dfrac{1}{x-1}$

$f(g(x))=\left(\dfrac{1}{x-1}\right)^2-1 = \boxed{\dfrac{1}{(x-1)^2}-1}$

$\qquad\qquad\qquad$ or $\boxed{\dfrac{1}{x^2-2x+1}-1}$

$g(f(x))=\dfrac{1}{(x^2-1)-1}=\boxed{\dfrac{1}{x^2-2}}$

3. $f(x)=\dfrac{1}{x-1};\ g(x)=(x+1)^2$

$f(g(x))=\dfrac{1}{(x+1)^2-1}=\dfrac{1}{x^2+2x+1-1}$

$\qquad\qquad = \boxed{\dfrac{1}{x^2+2x}}$

$g(f(x))=\boxed{\left(\dfrac{1}{x-1}+1\right)^2}$

2. $f(x)=x^3+2x^2+x-5;\ g(x)=x-2$

$f(g(x))=(x-2)^3+2(x-2)^2+(x-2)-5$

$\quad (x^3-6x^2+12x+8)+(2x^2-8x+8)+x-2-5$

$\quad \boxed{x^3-4x^2+5x-7}$

$g(f(x))=(x^3+2x^2+x-5)-2$

$\qquad = \boxed{x^3+2x^2+x-7}$

4. $f(x)=x^2-2;\ g(x)=\sqrt{x+1}\quad \sqrt{(x+1)^2}=x+1$

$f(g(x))=\left(\sqrt{x+1}\right)^2-2$

$\qquad = x+1-2$

$\qquad = \boxed{x-1}$

$g(f(x))=\sqrt{(x^2-2)+1}$

$\qquad = \boxed{\sqrt{x^2-1}}$

B. Find $(f\circ g)(3)$ and $(g\circ f)(-2)$.

5. $f(x)=2x-3;\ g(x)=x+1$

$f(g(3))\Rightarrow g(3)=3+1=4$

$\qquad\qquad f(4)=2(4)-3=\boxed{5}$

$g(f(-2))\Rightarrow f(3)=2(-2)-3=-7$

$\qquad\qquad g(-7)=-7+1=\boxed{-6}$

6. $f(x)=x^2-1;\ g(x)=2x-3$

$f(g(3))\Rightarrow g(3)=2(3)-3=3$

$\qquad\qquad f(3)=(3)^2-1=\boxed{8}$

$g(f(-2))\Rightarrow f(-2)=(-2)^2-1=3$

$\qquad\qquad g(3)=2(3)-3=\boxed{3}$

7. $f(x)=x^2;\ g(x)=\sqrt{x-1}$

$f(g(3))\Rightarrow g(3)=\sqrt{3-1}=\sqrt{2}$

$\qquad\qquad f(\sqrt{2})=(\sqrt{2})^2=\boxed{2}$

$g(f(-2))\Rightarrow f(-2)=(-2)^2=4$

$\qquad\qquad g(4)=\sqrt{4-1}=\boxed{\sqrt{3}}$

8. $f(x)=2x-3;\ g(x)=x^2-2x+3$

$f(g(3))\Rightarrow g(3)=(3)^2-2(3)+3=6$

$\qquad\qquad f(6)=2(6)-3=\boxed{9}$

$g(f(-2))\Rightarrow f(-2)=2(-2)-3=-7$

$\qquad\qquad g(-7)=(-7)^2-2(-7)+3=\boxed{66}$

C. Find two functions f and g such that $(f \circ g)(x) = h(x)$. Do not use $f(x) = x$ or $g(x) = x$.

9. $h(x) = (2x+1)^2$

$$f(x) = x^2$$
$$g(x) = 2x+1$$

10. $h(x) = (1-x)^3$

$$f(x) = x^3$$
$$g(x) = 1-x$$

11. $h(x) = (x+4)^2 + 2(x+4)$

$$f(x) = x^2 + 2x$$
$$g(x) = x+4$$