Honors Algebra II **Name** ______________________________

WS – Exponential and Logarithmic Application Problems Part 2

Set up each problem and show your work.

1. The population of Detroit was 1,514,063 in 1970 and 1,027,974 in 1990. Predict the population in the year 2010 if it grows according to the normal growth model.

2. The number N of bacteria in a culture is given by the model $N = 100e^{kt}$ where t is the time in hours. If N = 200 when t = 6, estimate the time required for the population to triple in size?

3. Rita puts \$5200 in a bank account at 7.2% annual interest, compounded quarterly. How long will it take to earn \$2400 interest?

4. A bone that originally contained 150 mg of carbon–14 now contains 85 mg of that isotope. Using the formula $A = A_0 2^{-t/k}$, where A is the present amount of the radioactive isotope, A_0 is the original amount of the isotope (in the same units as A), t is the time it takes to reduce the original amount of the isotope to its present amount, and k is the half-life of the isotope (measured in the same units as t), determine the age of the bone to the nearest 100 years. The half-life of carbon-14 is 5570 years.

5. The rate at which caffeine is eliminated from the bloodstream of an adult is about 15% per hour. An adult drinks a caffeinated soda, and the caffeine in his or her bloodstream reaches a peak level of 30 milligrams. Predict the amount, to the nearest tenth of a milligram, of caffeine remaining one hour after the peak level and 4 hours after the peak level. Let x = hours after the peak level.

6. In 1990 the cost of tuition at University of Georgia was \$4300. During the next 8 years the tuition rose 4% each year.
   i. Write an equation that gives the tuition y in dollars, t years after 1990.
   ii. What was the tuition in 1996?
   iii. When was the tuition approximately \$5000? Use your calculator!

7. The number, N, of bacteria present in a certain culture after t hours is given by the function $N = 10(2)^t$. Find N after 2 days.

8. The initial population of bacteria in a lab test is 400. The number of bacteria doubles every 30 minutes according to the normal growth model. Predict the population at the end of two hours and three hours.

9. Assume 775 bacteria triple every hour according to the normal growth model. Predict the population of bacteria after 2 hours and 4 hours.

10. A certain medication is eliminated from the bloodstream at a rate of 12% per hour according to the normal growth model. The medication reaches a peak level in the bloodstream of 40 milligrams. Predict the amount, to the nearest tenth of a milligram, of the medication remaining 2 hours after the peak level and 3 hours after the peak level.

1. $P = P_0 e^{kt}$

$1027974 = 1514063e^{20k}$

$0.679 = e^{20k}$

$\ln 0.679 = 20k$

$k = \frac{\ln 0.679}{20} = -.0194$

$P = 1514063e^{-.0194(40)}$

$P = 697943.6 \approx 698{,}000$ people

2. $200 = 100e^{6k}$

$2 = e^{6k}$

$\ln 2 = 6k$

$k = \frac{\ln 2}{6} = 0.116$

$3 = e^{.116t}$

$\ln 3 = .116t$

$t = \frac{\ln 3}{.116} = 9.51$ hours

3. $7600 = 5200(1 + \frac{.072}{4})^{4t}$

$\frac{19}{13} = 1.018^{4t}$

$\log_{1.018}(\frac{19}{13}) = 4t$

$t = \frac{\log_{1.018}(\frac{19}{13})}{4} = 5.32$ years

4. $75 = 150(2)^{-5570/k}$

$\frac{1}{2} = 2^{-5570/k}$

$\log_2 \frac{1}{2} = -\frac{5570}{k}$

$-1 = -\frac{5570}{k}$

$k = 5570$

$85 = 150(2)^{-t/5570}$

$\frac{17}{30} = (2)^{-t/5570}$

$\log_2(\frac{17}{30}) = \frac{-t}{5570}$

$t = -5570 \log_2(\frac{17}{30})$

$t = 4564.2$

$t \approx 4600$ years old

5. $P = 30(1-.15)^x$

$P = 30(.85)^1 = 25.5$ mg after 1 hour

$P = 30(.85)^4 = 15.66$ mg after 4 hours

6. $T(t) = 4300(1+.04)^t$

ii. $T(6) = 4300(1+.04)^6 = \$5440.87$

iii. $5000 = 4300(1.04)^t$

$\frac{50}{43} = 1.04^t$

$t = \log_{1.04}(\frac{50}{43}) = 3.85$ years, so approximately 1993.

7. $N = 10(2)^{48}$

$N = 2.81 \times 10^{15}$ bacteria

8. $P = 400(2)^{2t}$

$P = 400(2)^{2(2)} = 6400$ bacteria after 2 hours

$P = 400(2)^{2(3)} = 25,600$ bacteria after 3 hours

9. $3 = e^k$

$k = \ln 3 = 1.099$

$P = 775e^{(\ln 3)(2)} = 6975$ bacteria after 2 hours

$P = 775e^{(\ln 3)(4)} = 62,775$ bacteria after 4 hours

10. $.88 = e^k$

$\ln .88 = k = -.128$

$P = 40e^{(\ln .88)(2)} = 30.98$ mg after 2 hours

$P = 40e^{(\ln .88)(3)} = 27.26$ mg after 3 hours

#8 (normal growth model)

$4 = e^{1(k)}$

$\ln 4 = k$

$P = 400e^{(\ln 4)(2)} = 6400$ bacteria

$P = 400e^{(\ln 4)(3)} = 25,600$ bacteria