Honors Algebra II   Name _____

Unit 5B – Radical Functions   4 – WS – Inverses of Radical Functions

Find the inverse of each of the following functions. State the domain and range for the original function and its inverse.

1. $f(x) = \sqrt{x+4} - 3$

2. $g(x) = 2\sqrt[3]{x} - 5$

3. $f(x) = -\sqrt{2x+3} - 4$

4. $f(x) = -\sqrt{x-6} + 2$

5. $g(x) = \sqrt{x} + 1$

6. $h(x) = \sqrt[3]{x} - 10$
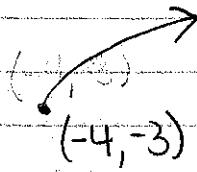
7. $f(x) = 4\sqrt{6-x} - 8$

8. $f(x) = 3\sqrt[3]{2x+1}$

9. $f(x) = -2\sqrt[3]{8-x}$

1. $f(x) = \sqrt{x+4} - 3$

   D: $[-4, \infty)$

   R: $[-3, \infty)$

   $(-4, -3)$

   $x = \sqrt{y+4} - 3$
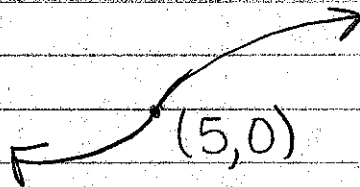
   $x + 3 = \sqrt{y+4}$

   $(x+3)^2 = y + 4$

   $\boxed{f^{-1}(x) = (x+3)^2 - 4}$

   $\boxed{D: [-3, \infty) \qquad R: [-4, \infty)}$

2. $g(x) = 2\sqrt[3]{x} - 5$

   D: $(-\infty, \infty)$

   R: $(-\infty, \infty)$

   $(5, 0)$

   $x = 2\sqrt[3]{y} - 5$

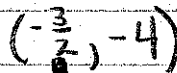 $\frac{1}{2}x = \sqrt[3]{y} - 5$

   $\frac{1}{8}x^3 = y - 5$

   $\boxed{g^{-1}(x) = \frac{1}{8}x^3 + 5}$

   $\boxed{D: (-\infty, \infty) \qquad R: (-\infty, \infty)}$

3. $f(x) = -\sqrt{2x+3} - 4$

   $\quad = -\sqrt{2\left(x+\frac{3}{2}\right)} - 4$

   D: $\left[-\frac{3}{2}, \infty\right) \qquad R: (-\infty, -4]$

   $\left(-\frac{3}{2}, -4\right)$

   $x = -\sqrt{2y+3} - 4$

   $x + 4 = -\sqrt{2y+3}$

   $-1(x+4) = \sqrt{2y+3}$

   $(x+4)^2 = 2y + 3$

   $(x+4)^2 - 3 = 2y$

   $\boxed{f^{-1}(x) = \frac{1}{2}(x+4)^2 - \frac{3}{2}}$

   $\boxed{D: (-\infty, -4] \qquad R: \left[-\frac{3}{2}, \infty\right)}$

4. $f(x) = -\sqrt{x-6} + 2$ (6,2)

D: $[6, \infty)$ R: $(-\infty, 2]$

$x = -\sqrt{y-6} + 2$

$x - 2 = -\sqrt{y-6}$

$-1(x-2) = \sqrt{y-6}$

$(x-2)^2 = y - 6$

$f^{-1}(x) = (x-2)^2 + 6$

D: $(-\infty, 2]$ R: $[6, \infty)$

5. $g(x) = \sqrt{x} + 1$

D: $[0, \infty)$ R: $[1, \infty)$ (0,1)

$x = \sqrt{y} + 1$

$x - 1 = \sqrt{y}$

$g^{-1}(x) = (x-1)^2$

D: $[1, \infty)$ R: $[0, \infty)$

6. $h(x) = \sqrt[3]{x-10}$

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ (10,0)

$x = \sqrt[3]{y-10}$

$x^3 = y - 10$

$h^{-1}(x) = x^3 + 10$

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

7. $f(x) = 4\sqrt{6-x} - 8$

$\quad = 4\sqrt{-1(x-6)} - 8$

D: $(-\infty, 6]$ R: $[-8, \infty)$

$(6, -8)$

$x = 4\sqrt{-1(y-6)} - 8$

$x + 8 = 4\sqrt{-1(y-6)}$

$\frac{1}{4}(x+8) = \sqrt{-1(y-6)}$

$\frac{1}{16}(x+8)^2 = -1(y-6)$

$-\frac{1}{16}(x+8)^2 = y - 6$

$f^{-1}(x) = -\frac{1}{16}(x+8)^2 + 6$

D: $[-8, \infty)$ R: $(-\infty, 6]$

8. $f(x) = 3\sqrt[3]{2x+1}$

$\quad = 3\sqrt[3]{2(x+\frac{1}{2})}$

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

$(-\frac{1}{2}, 0)$

$x = 3\sqrt[3]{2(y+\frac{1}{2})}$

$\frac{1}{3}x = \sqrt[3]{2(y+\frac{1}{2})}$

$\frac{1}{27}x^3 = 2(y+\frac{1}{2})$

$\frac{1}{54}x^3 = y + \frac{1}{2}$

$f^{-1}(x) = \frac{1}{54}x^3 - \frac{1}{2}$

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

9. $f(x) = -2\sqrt[3]{8-x}$

$\quad = -2\sqrt[3]{-1(x-8)}$

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

$(8, 0)$

$x = -2\sqrt[3]{-1(y-8)}$

$-\frac{1}{2}x = \sqrt[3]{-1(y-8)}$

$-\frac{1}{8}x^3 = -1(y-8)$

$\frac{1}{8}x^3 = y - 8$

$f^{-1}(x) = \frac{1}{8}x^3 + 8$

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$