

For each function:

- List the possible rational roots of f
- Use synthetic division and the possible roots to find a factor of the polynomial
- Factor the polynomial completely

1. $f(x) = x^3 + x^2 - 4x - 4$

2. $f(x) = -3x^3 + 20x^2 - 36x + 16$

3. $f(x) = -4x^3 + 15x^2 - 8x - 3$

4. $f(x) = 4x^4 - 17x^2 + 4$

5. $f(x) = 6x^3 - x^2 - 13x + 8$

6. $f(x) = x^4 - x^3 - 2x - 4$

7. $f(x) = 2x^4 - 7x^3 - 2x^2 - 7x - 4$

8. $f(x) = 2x^4 - 11x^3 - 6x^2 + 64x + 32$

WS - Rational Root Theorem and Factoring

1. $f(x) = x^3 + x^2 - 4x - 4$

a. $\frac{p=4}{1, 2, 4}$
 $\frac{q=1}{1}$

$\pm 1, \pm 2, \pm 4$

b.
$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & 0 \end{array} \Rightarrow (x+1)(x^2-4)$$

c. $f(x) = (x+1)(x+2)(x-2)$

2. $f(x) = -3x^3 + 20x^2 - 36x + 16 = -1(3x^3 - 20x^2 + 36x - 16)$

a. $\frac{p=16}{1, 2, 4, 8, 16}$

$\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}, \pm 8, \pm \frac{8}{3}, \pm 16, \pm \frac{16}{3}$

$\frac{q=3}{1, 3}$

$1, 3$

b.
$$\begin{array}{r|rrrr} 1 & 3 & -20 & 36 & -16 \\ & & 3 & -17 & \\ \hline & 3 & -17 & 19 & \end{array}$$

$$\begin{array}{r|rrrr} 2 & 3 & -20 & 36 & -16 \\ & & 6 & -28 & 16 \\ \hline & 3 & -14 & 8 & 0 \end{array} \Rightarrow (x-2)(3x^2-14x+8)$$

c.
$$\begin{aligned} f(x) &= (x-2)(3x^2-14x+8) \\ &= (x-2)(3x^2-2x-12x+8) \\ &= (x-2)(x-4)(3x-2) \end{aligned}$$

$$\begin{array}{r} 24 \\ -12 \quad -2 \\ -14 \end{array}$$

$$3. f(x) = -4x^3 + 15x^2 - 8x - 3 = -1(4x^3 - 15x^2 + 8x + 3)$$

$$a. \begin{array}{l} p = 3 \\ \hline 1, 3 \\ q = 4 \\ \hline 1, 2, 4 \end{array} \quad \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$$

$$b. \begin{array}{r} \underline{1} \quad 4 \quad -15 \quad 8 \quad 3 \\ \phantom{\underline{1}} \quad \quad 4 \quad -11 \quad -3 \\ \hline 4 \quad -11 \quad -3 \quad \underline{0} \end{array} \Rightarrow (x-1)(4x^2 - 11x - 3)$$

$$c. f(x) = (x-1)(4x^2 - 11x - 3) \quad \begin{array}{l} -12 \\ \hline -12 \quad 1 \\ \hline -11 \end{array}$$

$$= (x-1)(4x - 12x + 1x - 3)$$

$$= (x-1)(4x+1)(x-3)$$

$$4. f(x) = 4x^4 - 17x^2 + 4$$

$$a. \begin{array}{l} p = 4 \\ \hline 1, 2, 4 \\ q = 4 \\ \hline 1, 2, 4 \end{array} \quad \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm \frac{2}{2}, \pm \frac{2}{4}, \pm 4, \pm \frac{4}{2}, \pm \frac{4}{4}$$

$$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 4$$

$$b. \begin{array}{r} \underline{2} \quad 4 \quad 0 \quad -17 \quad 0 \quad 4 \\ \phantom{\underline{2}} \quad \quad 8 \quad 16 \quad -2 \quad -4 \\ \hline -2 \quad 4 \quad 8 \quad -1 \quad -2 \quad \underline{0} \\ \quad \quad -8 \quad 0 \quad 2 \\ \hline 4 \quad 0 \quad -1 \quad \underline{0} \end{array} \Rightarrow (x-2)(x+2)(4x^2-1)$$

$$c. f(x) = (x-2)(x+2)(2x-1)(2x+1)$$

5. $f(x) = 6x^3 - x^2 - 13x + 8$

a. $p = 8$

$1, 2, 4, 8$

$q = 6$

$1, 2, 3, 6$

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm 4, \pm \frac{4}{2}, \pm \frac{4}{3}, \pm 8, \pm \frac{8}{2}, \pm \frac{8}{3}, \pm \frac{8}{6}$

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}, \pm 8, \pm \frac{8}{3}$

b. $\begin{array}{r} \underline{1} \mid 6 \quad -1 \quad -13 \quad 8 \\ \phantom{\underline{1} \mid} 6 \quad 5 \quad -8 \\ \hline 6 \quad 5 \quad -8 \quad 0 \end{array} \Rightarrow (x-1)(6x^2+5x-8)$

c. $f(x) = (x-1)(6x^2+5x-8)$
not factorable

$\frac{-48}{5}$

$(5)^2 - 4(6)(-8) = 217$

6. $f(x) = x^4 - x^3 - 2x - 4$

a. $p = 4$

$1, 2, 4$

$q = 1$

1

$\pm 1, \pm 2, \pm 4$

b. $\begin{array}{r} \underline{1} \mid 1 \quad -1 \quad 0 \quad -2 \quad -4 \\ \phantom{\underline{1} \mid} \quad 1 \quad 0 \quad 0 \quad -2 \\ \hline 1 \quad 0 \quad 0 \quad -2 \quad -6 \\ -\underline{1} \mid 1 \quad -1 \quad 0 \quad -2 \quad -4 \\ \phantom{-\underline{1} \mid} \quad -1 \quad 2 \quad -2 \quad 4 \\ \hline 1 \quad -2 \quad 2 \quad -4 \quad 0 \end{array} \Rightarrow (x+1)(x^3-2x^2+2x-4)$

c. $f(x) = (x+1)(x^3-2x^2+2x-4)$
 $= (x+1)(x^2+2)(x-2)$

$$7. f(x) = 2x^4 - 7x^3 - 2x^2 - 7x - 4$$

$$p = 4$$

$$1, 2, 4$$

$$q = 2$$

$$1, 2$$

$$\pm 1, \pm \frac{1}{2}, \pm 2, \pm \frac{2}{2}, \pm 4, \pm \frac{4}{2}$$

$$\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4$$

$$b. \quad 1) \begin{array}{r} 2 \quad -7 \quad -2 \quad -7 \quad -4 \\ \underline{2 \quad -5 \quad -7} \\ 2 \quad -5 \quad -7 \end{array}$$

$$-1) \begin{array}{r} 2 \quad -7 \quad -2 \quad -7 \quad -4 \\ \underline{-2 \quad 9 \quad -7} \\ 2 \quad -9 \quad 7 \end{array}$$

$$\frac{1}{2}) \begin{array}{r} 2 \quad -7 \quad -2 \quad -7 \quad -4 \\ \underline{1 \quad -3} \\ 2 \quad -6 \quad -5 \end{array}$$

$$-2) \begin{array}{r} 2 \quad -7 \quad -2 \quad -7 \quad -4 \\ \underline{-4 \quad 22 \quad -40} \\ 2 \quad -11 \quad 20 \end{array}$$

$$2) \begin{array}{r} 2 \quad -7 \quad -2 \quad -7 \quad -4 \\ \underline{4 \quad -6 \quad 16} \\ 2 \quad -3 \quad -8 \quad 9 \end{array}$$

$$-\frac{1}{2}) \begin{array}{r} 2 \quad -7 \quad -2 \quad -7 \quad -4 \\ \underline{-1 \quad 4 \quad -1 \quad 4} \\ 2 \quad -8 \quad 2 \quad -8 \quad 0 \end{array}$$

$$2 \\ (2x+1)(x^3-4x^2+x-4)$$

$$c. f(x) = (2x+1)(x^3-4x^2+x-4) \\ = (2x+1)(x^2+1)(x-4)$$

8. $f(x) = 2x^4 - 11x^3 - 6x^2 + 64x + 32$

a. $p = 32$

$1, 2, 4, 8, 16, 32$

$q = 2$

$\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

$1, 2$

b. $1) \begin{array}{r} 2 \quad -11 \quad -6 \quad 64 \quad 32 \\ \underline{2 \quad -9 \quad -15} \end{array}$

~~$2 \quad -9 \quad -15$~~

$-1) \begin{array}{r} 2 \quad -11 \quad -6 \quad 64 \quad 32 \\ \underline{-2 \quad 13 \quad -7} \end{array}$

~~$2 \quad -13 \quad 7$~~

$2) \begin{array}{r} 2 \quad -11 \quad -6 \quad 64 \quad 32 \\ \underline{4 \quad -14 \quad 40 \quad 48} \end{array}$

~~$2 \quad -7 \quad -20 \quad 24$~~

$-2) \begin{array}{r} 2 \quad -11 \quad -6 \quad 64 \quad 32 \\ \underline{-4 \quad 30 \quad -48 \quad -32} \end{array}$

$-2) \begin{array}{r} 2 \quad -15 \quad 24 \quad 16 \quad 0 \\ \underline{-4 \quad 38} \end{array}$

~~$2 \quad -19$~~

$\frac{1}{2}) \begin{array}{r} 2 \quad -15 \quad 24 \quad 16 \\ \underline{1 \quad -7} \end{array}$

~~$2 \quad -14 \quad 17$~~

$-\frac{1}{2}) \begin{array}{r} 2 \quad -15 \quad 24 \quad 16 \\ \underline{-1 \quad 8 \quad -16} \end{array}$

$2 \quad -16 \quad 32 \quad 0$

2

$(x+2)(2x+1)(x^2-8x+16)$

c. $f(x) = (x+2)(2x+1)(x-4)^2$