1. $(\sqrt{x+2} = 5)^2$

$x + 2 = 25$

$\boxed{x = 23}$ ✓

2. $(\sqrt{x+4} = 3\sqrt{x})^2$

$x + 4 = 9x$

$4 = 8x$

$\boxed{\frac{1}{2} = x}$ ✓

3. $(3\sqrt[3]{x} = \sqrt[3]{7x+5})^3$

$27x = 7x + 5$

$20x = 5$

$\boxed{x = \frac{1}{4}}$ ✓

4. $(\sqrt{-14x+2} = x - 3)^2$

$-14x + 2 = x^2 - 6x + 9$

$0 = x^2 + 8x + 7$

$0 = (x+7)(x+1)$

$x = \cancel{-7}, \cancel{-1}$

$\boxed{\text{no solution}}$

5. $4(x-12)^{\frac{1}{3}} = -16$

$((x-12)^{\frac{1}{3}} = -4)^3$

$x - 12 = -64$

$\boxed{x = -52}$

6. $\sqrt[3]{4x+1} - 5 = 0$

$(\sqrt[3]{4x+1} = 5)^3$

$4x + 1 = 125$

$4x = 124$

$\boxed{x = 31}$

7. $3\sqrt{x-11} = 18$

$(\sqrt{x-11} = 6)^2$

$x - 11 = 36$

$\boxed{x = 47}$

8. $(\sqrt[4]{10x+11} = 3)^4$

$10x + 11 = 81$

$10x = 70$

$\boxed{x = 7}$

9. $(x + 2 = \sqrt{3x+6})^2$

$x^2 + 4x + 4 = 3x + 6$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\boxed{x = -2, 1}$

10. $((10x-25)^{\frac{1}{2}} = x)^2$

$10x - 25 = x^2$

$0 = x^2 - 10x + 25$

$0 = (x-5)^2$

$\boxed{x = 5}$

11. $5(6x+1)^{\frac{1}{4}} = 10$

$((6x+1)^{\frac{1}{4}} = 2)^4$

$6x + 1 = 16$

$6x = 15$

$\boxed{x = \frac{5}{2}}$

12. $4(7x+18)^{\frac{1}{2}} = 4x$

$((7x+18)^{\frac{1}{2}} = x)^2$

$7x + 18 = x^2$

$0 = x^2 - 7x - 18$

$0 = (x-9)(x+2)$

$\boxed{x = 9, \cancel{-2}}$

13. $(\sqrt{x-3} = \sqrt{x+15} - 2)^2$

$x - 3 = x + 15 - 4\sqrt{x+15} + 4$

$x - 3 = x + 19 - 4\sqrt{x+15}$

$-22 = -4\sqrt{x+15}$

$\frac{11}{2} = \sqrt{x+15}$

$\frac{121}{4} = x + 15$

$\boxed{x = \frac{61}{4}}$

14. $\sqrt{x+16} = x - \sqrt{x+7}$

$x + 16 = x^2 - 2x\sqrt{x+7} + x + 7$

$(2x\sqrt{x+7} = x^2 - 9)^2$

$4x^2(x+7) = x^4 - 18x^2 + 81$

$4x^3 + 28x^2 = x^4 - 18x^2 + 81$

$0 = x^4 - 4x^3 - 46x^2 + 81$

9	1	-4	-46	0	81
		9	45	-9	-81
	1	5	-1	-9	0

$\boxed{x = 9}$

15. $\sqrt{x-3} - \sqrt{x-2} = 1$

$(\sqrt{x-3} = 1 + \sqrt{x-2})^2$

$x-3 = 1 + 2\sqrt{x-2} + x-2$

$-2 = 2\sqrt{x-2}$

no solution

16. $\sqrt{\sqrt{x-3}} = \sqrt{x-15}$

$\sqrt{x-3} = x-15$

$x-3 = x^2-30x+225$

$0 = x^2-31x+228$

$\frac{31 \pm \sqrt{961-4(228)}}{2}$

$\frac{31 \pm \sqrt{49}}{2} = \frac{31 \pm 7}{2} = 19, \cancel{12}$ ✓

17. $\sqrt{x^2-7x+12} - x = x-6$

$\sqrt{x^2-7x+12} = 2x-6$

$x^2-7x+12 = 4x^2-24x+36$

$0 = 3x^2-17x+24$ $\Rightarrow x^2-17x+72$

$0 = (3x-8)(x-3)$ $(x-\frac{8}{3})(x-\frac{9}{3})$

$x = \cancel{\frac{8}{3}}, 3$

18. $(\sqrt{4x+5} \leq 3)^2$

$4x+5 \leq 9$

$4x \leq 4$

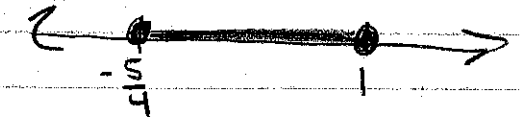
$x \leq 1$

$4x+5 \geq 0$

$4x \geq -5$

$x \geq -\frac{5}{4}$

$[-\frac{5}{4}, 1]$



19. $\sqrt{x-4}+3>9$

$\sqrt{x-4}>6$

$x-4>36$

$x>40$

$(40,\infty)$

$x-4\geq 0$

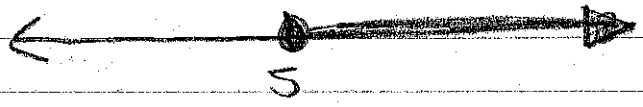
~~$x\geq 4$~~

20. $\sqrt[3]{x+3}\geq 2$

$x+3\geq 8$

$x\geq 5$

$[5,\infty)$



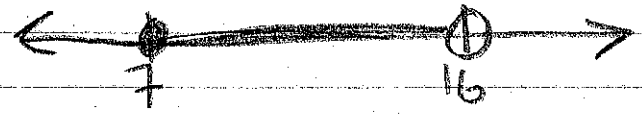
21. $\sqrt{x-7}+9<12$

$\sqrt{x-7}<3$

$x-7<9$

$x<16$

$[7,16)$



$x-7\geq 0$

$x\geq 7$

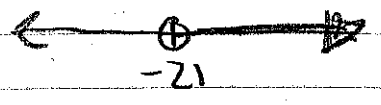
22. $\sqrt[3]{x-6}+7>4$

$\sqrt[3]{x-6}>-3$

$x-6>-27$

$x>-21$

$(-21,\infty)$



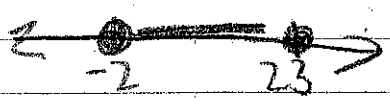
23. $\sqrt{x+2}-1\leq 4$

$\sqrt{x+2}\leq 5$

$x+2\leq 25$

$x\leq 23$

$[-2,23]$



$x+2\geq 0$

$x\geq -2$