

Name _____

Algebra II

Unit 4B Systems of Polynomials WS

Solve each system of equations algebraically.

1. $y = x^3 - 3x^2 + 3$
 $y = -2x + 3$

2. $f(x) = 3x$
 $g(x) = x^3 - x$

3. $y = x^3 + 44x$
 $y = 48 + 12x^2$

4. $f(x) = x^4 + 2x^3 - 35x^2$
 $g(x) = 72x + 36$

For the following word problems, use algebraic methods to solve for

5. A rectangle has a length of $(x^2 + 3x)$ and a width of $(x^2 + 12x - 28)$.
- a. Write a function for the area of the rectangle. If the area is 1008 ft^3 , what is the length and width of the rectangle?
6. A rectangular prism with length $(8 - x)$, width $(x + 7)$ and height $(x - 2)$.
- a. Write a function $V(x)$ in standard form representing the volume of the prism.
- b. What are the dimensions of the prism if the volume is 108 cubic units?
- c. What are the dimensions of the prism if the volume is 50 cubic units?

$$1. \quad y = x^3 - 3x^2 + 3$$

$$y = -2x + 3$$

$$x^3 - 3x^2 + 3 = -2x + 3$$

$$x^3 - 3x^2 + 2x = 0$$

$$x(x^2 - 3x + 2) = 0$$

$$x(x-2)(x-1) = 0$$

$$x = 0, 1, 2$$

$$y = -2(0) + 3 = 3$$

$$y = -2(1) + 3 = 1$$

$$y = -2(2) + 3 = -1$$

$(0, 3)$
$(1, 1)$
$(2, -1)$

$$2. \quad f(x) = 3x$$

$$g(x) = x^3 - x$$

$$3x = x^3 - x$$

$$0 = x^3 - 4x$$

$$0 = x(x^2 - 4)$$

$$0 = x(x-2)(x+2)$$

$$x = 0, 2, -2$$

$$y = 3(0) = 0$$

$$y = 3(2) = 6$$

$$y = 3(-2) = -6$$

$(0, 0)$
$(2, 6)$
$(-2, -6)$

$$3. \quad y = x^3 + 44x$$

$$y = 48 + 12x^2$$

$$x^3 + 44x = 48 + 12x^2$$

$$x^3 - 12x^2 + 44x - 48 = 0$$

~~$$\begin{array}{r|rrrr} 1 & 1 & -12 & 44 & -48 \\ & & 1 & -11 & 33 \\ \hline & 1 & -11 & 33 & -15 \end{array}$$~~

$$\begin{array}{r|rrrr} 2 & 1 & -12 & 44 & -48 \\ & & 2 & -20 & 48 \\ \hline & 1 & -10 & 24 & 0 \end{array}$$

$$(x-2)(x^2 - 10x + 24) = 0$$

$$(x-2)(x-4)(x-6) = 0$$

$$x = 2, 4, 6$$

$$y = (2)^3 + 44(2) = 96$$

$$y = (4)^3 + 44(4) = 240$$

$$y = (6)^3 + 44(6) = 480$$

$(2, 96)$	$(4, 240)$	$(6, 480)$
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$$4. f(x) = x^4 + 2x^3 - 35x^2$$

$$g(x) = 72x + 36$$

$$x^4 + 2x^3 - 35x^2 = 72x + 36$$

$$x^4 + 2x^3 - 35x^2 - 72x - 36 = 0$$

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -35 & -72 & -36 \\ & & 1 & 3 & -82 & -104 \\ \hline & 1 & 3 & -32 & -104 & -140 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 2 & -35 & -72 & -36 \\ & & -1 & -1 & 36 & 36 \\ \hline & 1 & 1 & -36 & -36 & 0 \end{array}$$

$$(x+1)((x^3 + x^2 - 36x - 36)) = 0$$

$$(x+1)(x^2 - 36)(x+1) = 0$$

$$(x+1)^2(x-6)(x+6) = 0$$

$$x = -1 \quad x = 6 \quad x = -6$$

M. 2

$$g(-1) = 72(-1) + 36 = -36$$

$$g(6) = 72(6) + 36 = 468$$

$$g(-6) = 72(-6) + 36 = -396$$

$$\begin{array}{l} (-1, -36) \\ (6, 468) \\ (-6, -396) \end{array}$$

$$5. A(x) = (x^2 + 3x)(x^2 + 12x - 28)$$

$$1008 = x^4 + 15x^3 + 8x^2 - 84x$$

$$0 = x^4 + 15x^3 + 8x^2 - 84x - 1008$$

$$x^4 - 15x^3 + 8x^2 + 84x - 1008$$

~~$$\begin{array}{r} \times 1 \quad +15 \quad 8 \quad -84 \quad -1008 \\ \underline{ } \\ \end{array}$$~~

~~$$ $$~~

~~$$ $$~~

~~$$\times 1 \quad +15 \quad 8 \quad -84 \quad -1008$$~~

~~$$ $$~~

~~$$ $$~~

~~$$\times 1 \quad +15 \quad 8 \quad -84 \quad -1008$$~~

~~$$ $$~~

~~$$ $$~~

$$4 \mid 1 \quad 15 \quad 8 \quad -84 \quad -1008$$

$$ $$

$$ $$

$$x = 4$$

$$\text{Length} = (4)^2 + 3(4) = 28 \text{ ft}$$

$$\text{Width} = (4)^2 + 12(4) - 28 = 36 \text{ ft}$$

	<u>Max +</u>	<u>Max -</u>
	1	3

$$\begin{aligned}
 6. \quad V(x) &= (8-x)(x+7)(x-2) \\
 V(x) &= (8-x)(x^2+5x-14) \\
 V(x) &= -x^3+3x^2+54x-112
 \end{aligned}$$

Domain

$$\begin{aligned}
 8-x > 0 & \quad x+7 > 0 & \quad x-2 > 0 \\
 8 > x & \quad x > -7 & \quad x > 2 \\
 2 < x < 8
 \end{aligned}$$

$$\begin{aligned}
 b. \quad 108 &= -x^3+3x^2+54x-112 \\
 0 &= -x^3+3x^2+54x-220
 \end{aligned}$$

$$\begin{array}{r}
 \begin{array}{cccc}
 \cancel{2} & -1 & 3 & 54 & -220 \\
 & \cancel{-3} & 0 & 162 & \\
 \hline
 & -1 & 0 & 54 & -58 \\
 \cancel{4} & -1 & 3 & 54 & -220 \\
 & \cancel{-4} & \cancel{-4} & 200 & \\
 \hline
 & -1 & -1 & 50 & -20 \\
 \cancel{5} & -1 & 3 & 54 & -220 \\
 & \cancel{-5} & \cancel{-10} & 220 & \\
 \hline
 & -1 & -2 & 44 & 0
 \end{array}
 \end{array}$$

$$x = 5$$

length = $8-5 = 3$ units
 width = $5+7 = 12$ units
 height = $5-2 = 3$ units

$$\begin{aligned}
 c. \quad 50 &= -x^3+3x^2+54x-112 \\
 0 &= (-x^3+3x^2)+(54x-162) \\
 0 &= -1(x^3-3x^2-54x+162) \\
 0 &= -1(x^2-54)(x-3) \\
 x &= 3 \qquad x = \sqrt{54} \qquad x = -\sqrt{54} \\
 & \qquad \qquad \qquad = 3\sqrt{6} \qquad \qquad \qquad \cancel{x = -3\sqrt{6}}
 \end{aligned}$$

length = 5 units
 width = 10 units
 height = 1 unit

length = $8-3\sqrt{6}$ units $\approx .65$ units
 width = $3\sqrt{6}+7$ units ≈ 14.35 units
 height = $3\sqrt{6}-2$ units ≈ 5.35 units