Warm-up: Solve the following by change-of-base

1) $\left(\frac{1}{16}\right)^{2-x} \geq 8^{x+5}$ 2) $\left(\frac{1}{243}\right)^{2x-3} + 5 < 32$

Vocabulary

Transformation - an alteration in the function rule for the parent graph

Vertical Translation - if $y = f(x)$, then $y = f(x) + k$ yields a vertical shift of the graph of f.

- The shift is k units up for $k > 0$ and $|k|$ units down if $k < 0$.

Horizontal Translation if $y = f(x)$, then $y = f(x - h)$ yields a horizontal shift of the graph of f.

- The shift is h units to the right for $h > 0$ and $|h|$ units to the left for $h < 0$.

Vertical Stretch and Vertical Compression - if $y = f(x)$, then $y = af(x)$ yields a vertical stretch or vertical compression of the graph of f.

- If $|a| > 1$, the graph is stretched vertically by a factor of a.
- If $0 < |a| < 1$, the graph is compressed vertically by a factor of $|a|$

Horizontal Stretch and Horizontal Compression - if $y = f(x)$, then $y = f(bx)$ yields a horizontal stretch or horizontal compression of the graph of f.

- If $|b| > 1$, the graph is compressed horizontally by a factor of $\left|\frac{1}{b}\right|$.
- If $|b| < 1$, the graph is stretched horizontally by a factor of $\left|\frac{1}{b}\right|$.

Reflections -

- If $y = f(x)$, then $y = - f(x)$ yields a reflection of the graph of f across the x - axis.
- If $y = f(x)$, then $y = f(- x)$ yields a reflection of the graph of f across the y - axis.

Exponential Growth & Decay

EQ: How do you graph exponential growth/ decay and what are the graphs characteristics?

Standards:

MCC9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

MCC9-12.A.SSE.3c Use the properties of exponents to transform expressions for exponential functions.

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MCC9-12.F.IF.7 Graph exponential and logarithmic functions, showing intercepts and end behavior.

MCC9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MCC9-12.F.IF.8b Use the properties of exponents to interpret expressions for exponential functions

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given verbal description of the relationship. Key features include: intercepts, intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behaviors; and periodicity.

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions from them.

What am I learning today?

How to operate with and describe the characteristics of exponential functions

How will I show that I learned it?

Graph an exponential function by its transformations and rewrite in equivalent forms

Exponential Growth and Decay

· reflect over y-axis (x)
· h. dilate (x)

v. shift (y)

$$f(x) = -a \cdot B^{b(x-h)} + k$$

reflect over x-axis (y)

v. dilate (y)

h. shift (x)

if B > 1 ... growth

0 < B < 1 ... decay

Reflections over y-axis...

Turns **growth** into **decay** and **decay** into **growth**.

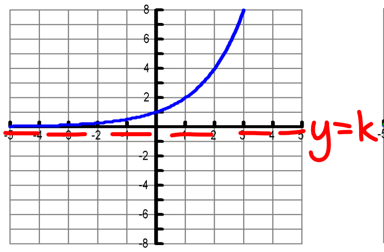
$f(x) = 2^{-x} = \frac{1}{2}^{x}$

$g(x) = (1/3)^{-x} = 3^{x}$

$h(x) = (3/2)^{\overset{-x+4}{4-x}} = \left(\frac{3}{2}\right)^{-1(x-4)} = \left(\frac{2}{3}\right)^{x-4}$

Exponential Growth Graphs

$f(x) = ab^{x-h} + k$



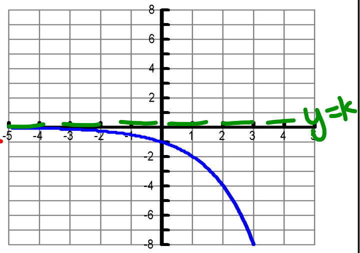
D: $(-\infty,\infty)$ R: (k,∞)

asymptote: $y=k$

intervals: inc: $(-\infty,\infty)$

end behaviors:
As $x\to-\infty$, $f(x)\to$ k
As $x\to\infty$, $f(x)\to\infty$

$f(x) = -ab^{x-h} + k$



D: $(-\infty,\infty)$ R: $(-\infty,k)$

asymptote: $y=k$

intervals: dec: $(-\infty,\infty)$

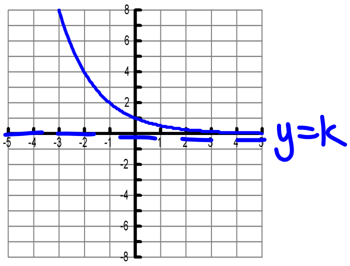
end behaviors:
As $x\to-\infty$, $f(x)\to$ k
As $x\to\infty$, $f(x)\to-\infty$

x-intercept: sometimes
depends on graph (use calculator to find it)

y-intercept: always!
plug in zero for x and solve

Exponential Decay Graphs

$f(x) = ab^{x-h} + k$



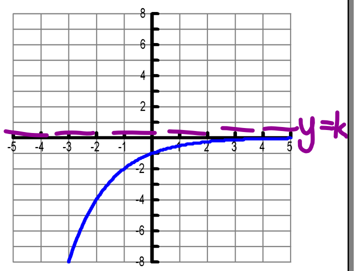
D: $(-\infty,\infty)$ R: (k,∞)

asymptote: $y=k$

intervals: dec: $(-\infty,\infty)$

end behaviors:
As $x\to-\infty$, $f(x)\to\infty$
As $x\to\infty$, $f(x)\to$ k

$f(x) = -ab^{x-h} + k$



D: $(-\infty,\infty)$ R: $(-\infty,k)$

asymptote: $y=k$

intervals: inc: $(-\infty,\infty)$

end behaviors:
As $x\to-\infty$, $f(x)\to-\infty$
As $x\to\infty$, $f(x)\to$ k

x-intercept: sometimes
depends on graph (use calculator to find it)

y-intercept: always!
plug in zero for x and solve

$$f(x) = a\,B^{b(x-h)} + k$$

Find the parent function and type of exponential.

Discuss all transformations.

Then, rewrite if possible.

$f(x) = 4(1/2)^{x-3} + 6$

Parent: $y = \frac{1}{2}^{x}$ Type: decay

Transformations:

- v. stretch of 4 (y)
- shift right 3 (x)
- shift up 6 (y)

$x+3 \mid 4y+6$

Rewrite: $4\left(\frac{1}{2}\right)^{x-3}+6$

$= 2^2\left(\frac{1}{2}\right)^{x-3}+6$

$= \frac{1}{2}^{-2}\cdot\left(\frac{1}{2}\right)^{x-3}+6$

$\boxed{f(x) = \left(\frac{1}{2}\right)^{x-5}+6}$

$$f(x) = a\,B^{b(x-h)} + k$$

Find the parent function and type of exponential.

Discuss all transformations.

Then, rewrite if possible.

$g(x) = 27(1/9)^{\overset{-x+2}{2-x}} = 27\left(\frac{1}{9}\right)^{-1(x-2)}$

Parent: $y = \frac{1}{9}^{x}$ Type: decay

Transformations:

- v. stretch by 27 (y)
- reflect over y-axis (x)
- shift right 2 (x)

$-x+2 \mid 27y$

Rewrite:

$= 27(9)^{x-2}$

$= 3^3(3^2)^{x-2}$

$= 3^3(3)^{2x-4}$

$\boxed{g(x) = (3)^{2x-1}}$

Are the following examples of exponential growth or decay? Without graphing, what are the characteristics besides x-intercepts? Rewrite the function if possible.

1) $y = 4 \bullet 2^{x-3} - 1$

Parent Graph: $y = 2^x$

- v. stretch by 4 (y)
- shift right 3 (x)
- shift down 1 (y)

$y = -1$

type: growth

$x+3, 4y-1$

domain: $(-\infty, \infty)$

range: $(-1, \infty)$

$4 \cdot 2^{-3} - 1 = 4 \cdot \frac{1}{8} - 1 = \frac{1}{2} - 1$

asymptote: $y = -1$

y - int: $(0, -\frac{1}{2})$

interval of increasing: $(-\infty, \infty)$

end behaviors: As $x \to -\infty$, $y \to -1$
As $x \to \infty$, $y \to \infty$

Rewrite of function:

$$y = 4 \bullet 2^{x-3} - 1$$

$$= 2^2 \cdot 2^{x-3} - 1$$

$$\boxed{y = 2^{x-1} - 1}$$

2) $y = -\left(\frac{1}{5}\right)^x - 4$

$y = -4$

Parent Graph: $y = \frac{1}{5}^x$

- reflect over x-axis (y)
- shift down 4 (y)

$x, -y-4$

type: decay

domain: $(-\infty, \infty)$

range: $(-\infty, -4)$

asymptote: $y = -4$

y - int: $(0, -5)$

$-\left(\frac{1}{5}\right)^0 - 4$

$-1 - 4$

interval of increasing: $(-\infty, \infty)$

end behaviors: As $x \to -\infty$, $y \to -\infty$
As $x \to \infty$, $y \to -4$

Rewrite of function:

$$y = -\left(\frac{1}{5}\right)^x - 4$$

none

3) $y = -\frac{1}{3} \bullet 3^x + 4$

Parent Graph: $y = 3^x$

$y=4$

· reflect over x-axis (y)
· v. compress by $\frac{1}{3}$ (y)
· shift up 4 (y)

$x \mid \frac{1}{3}y+4$

type: growth

domain: $(-\infty, \infty)$ range: $(-\infty, 4)$

asymptote: $y=4$ y - int: $(0, \frac{11}{3})$

$-\frac{1}{3} \cdot 3^0 + 4$

$-\frac{1}{3} + \frac{12}{3}$

interval of decreasing: $(-\infty, \infty)$

end behaviors: As $x \to -\infty$, $y \to 4$
As $x \to \infty$, $y \to -\infty$

Rewrite of function: $y = -\frac{1}{3} \bullet 3^x + 4$

$= -3^{-1} \cdot 3^x + 4$

$\boxed{y = -3^{x-1} + 4}$

4) $y = \frac{1}{8}\left(\frac{1}{2}\right)^{x+6}$

$y=0$

Parent Graph: $y = \frac{1}{2}^x$

· v. comp. by $\frac{1}{8}$ (y)
· shift left 6 (x) $x-6 \mid \frac{1}{8}y$

type: decay

domain: $(-\infty, \infty)$ range: $(0, \infty)$

asymptote: $y=0$ y - int: $(0, \frac{1}{512})$

$\frac{1}{8} \cdot \frac{1}{2}^6 = \frac{1}{8} \cdot \frac{1}{64}$

interval of decreasing: $(-\infty, \infty)$

end behaviors: As $x \to -\infty$, $y \to \infty$
As $x \to \infty$, $y \to 0$

Rewrite of function: $y = \frac{1}{8}\left(\frac{1}{2}\right)^{x+6}$

$= \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^{x+6}$

$\boxed{y = \left(\frac{1}{2}\right)^{x+9}}$

5) $y = \frac{1}{4} \bullet 8^{x+2} - 3$ $y=-3$

Parent Graph: $y = 8^x$

· v. comp. by $\frac{1}{4}$ (y)
· shift left 2 (x)
· shift down 3 (y)

$x-2 \mid \frac{1}{4}y-3$

type: growth

domain: $(-\infty, \infty)$ range: $(-3, \infty)$

asymptote: $y=-3$ y - int: $(0, 13)$

$\frac{1}{4}(8)^2 - 3$
$= \frac{1}{4} \cdot 64 - 3$
$= 16 - 3$

interval of in creasing: $(-\infty, \infty)$

end behaviors: As $x \to -\infty$, $y \to -3$
As $x \to \infty$, $y \to \infty$

Rewrite of function: $y = \frac{1}{4} \bullet 8^{x+2} - 3$

$= 2^{-2} \cdot (2^3)^{x+2} - 3$
$= 2^{-2} \cdot 2^{3x+6} - 3$
$\boxed{y = 2^{3x+4} - 3}$

6) $y = 3(4)^{5-x} + 1$ $y=1$

$-x+5$

$= 3(4)^{-1(x-5)} + 1$

Parent Graph: $y = 4^x$

· v. stretch by 3 (y)
· reflect over y-axis (x)
· shift right 5 (x) + up 1 (y)

$-x+5 \mid 3y+1$

type: growth

domain: $(-\infty, \infty)$ range: $(1, \infty)$

asymptote: $y=1$ y - int: $(0, 3073)$

$3(4)^5 + 1$
$= 3(1024) + 1$
$= 3072 + 1$

interval of de creasing: $(-\infty, \infty)$

end behaviors: As $x \to -\infty$, $y \to \infty$
As $x \to \infty$, $y \to 1$

Summary of Transformations

Transformation of $f(x)$	Transformed Function
Vertical shift of k units up	$y = f(x) + k$, for $k > 0$
Vertical shift of $\lvert k \rvert$ units down	$y = f(x) + k$, for $k < 0$
Horizontal shift of h units right	$y = f(x - h)$, for $h > 0$
Horizontal shift of $\lvert h \rvert$ units left	$y = f(x - h)$, for $h < 0$
Vertical stretch by a factor of $\lvert a \rvert$	$y = af(x)$, where $\lvert a \rvert > 1$
Vertical compression by a factor of $\lvert a \rvert$	$y = af(x)$, where $0 < \lvert a \rvert < 1$
Horizontal stretch by a factor of $\lvert 1/b \rvert$	$y = f(bx)$, where $0 < \lvert b \rvert < 1$
Horizontal compression by a factor of $\lvert 1/b \rvert$	$y=f(bx)$, where $\lvert b \rvert > 1$
Reflection across the x-axis	$y = - f(x)$
Reflection across the y-axis	$y = f(-x)$

Describe the transformations to the function $f(x) = 2^x$.

1. $g(x) = 2^{x + 3} - 7$
   - shift left 3 (x)
   - shift down 7 (y)

   $x-3 \mid y-7$

2. $g(x) = 3(2)^{x - 3}$
   - v. stretch by 3 (y)
   - shift right 3 (x)

   $x+3 \mid 3y$

3. $g(x) = 1/3(2)^x + 5$
   - v. comp. by 1/3 (y)
   - shift up 5 (y)

   $x \mid \frac{1}{3}y+5$

4. $g(x) = -(2)^{3(x - 1)} - 2$
   - reflect over x-axis (y)
   - h. comp. by 1/3 (x)
   - shift right 1 (x)
   - shift down 2 (y)

   $\frac{1}{3}x+1 \mid -y-2$

5. $g(x) = 2^{1/2x + 3}$

   $= 2^{\frac{1}{2}(x+6)}$
   - h. stretch by 2 (x)
   - shift left 6 (x)

   $2x-6 \mid y$

6. $g(x) = 2^{-2(x + 4)}$
   - reflect over y-axis (y)
   - h. comp by 1/2 (x)
   - shift left 4 (x)

   $-\frac{1}{2}x-4 \mid y$