Geometric Series

Standard:
9-12.A.SSE.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

What am I learning today?

How to evaluate a geometric series in sigma notation

How will I show that I learned it?

Expand a series from sigma notation and evaluate it.

Geometric Sequences

Sequences that are created by MULTIPLYING the same value. We call this value the COMMON RATIO. When graphed, it looks like an EXPONENTIAL FUNCTION with the ratio related to the base.

Example: 1, 3, 9, 27, 81

Geometric Sequence

- **The ratio of a term to its previous term is constant.**
- **This number that you multiply by is called the common ratio (r).**
- **The common ratio is $r = \frac{a_2}{a_1}$**
- **The formula is $a_n = a_1(r)^{n-1}$**

$(1, \#)$

$y = a(b)^x$

$(0, \#)$

$a_n = a_5(r)^{n-5}$

Write the equation for the following geometric sequences.

Ex. A 8, 12, 18, 27, 40.5, ...

$a_1 = 8 \quad r = \frac{12}{8} = \frac{3}{2}$ or 1.5

$a_n = 8\left(\frac{3}{2}\right)^{n-1}$

Ex. B 10, 11, 12.1, 13.31, 14.641, ...

$a_1 = 10 \quad r = \frac{11}{10}$ or 1.1

$a_n = 10\left(\frac{11}{10}\right)^{n-1}$

Example: HARDEST Questions!

1) Two terms of a geometric sequence are $a_2 = -4$ and $a_6 = -1024$. Write a rule for the nth term.

a) Fill in so the higher term is your a_n and your "a_1" is your lower to find the common ratio. If you must take an even root remember you have ± r so you have two different possibilities for the "rule".

$a_2 = -4, a_6 = -1024$

$a_n = a_2(r)^{n-2}$

$a_6 = a_2(r)^{6-2}$

$-1024 = -4(r)^4$

$r^4 = 256$

$r = \sqrt[4]{256}$

$r = \pm 4$

b) Use your "r"(s) and your higher a_n to find a_1 & write the new rule for the nth term.

$r = 4, a_2 = -4$

$a_1 \cdot r = a_2$

$a_1 = \frac{a_2}{r} = \frac{-4}{4} = -1$

$\boxed{a_n = -1(-4)^{n-1}}$

$r = -4, a_2 = -4$

$a_1 = \frac{-4}{-4} = 1$

$\boxed{a_n = 1(-4)^{n-1}}$

Example: HARDEST Questions!

1) Two terms of a geometric sequence are $a_3 = 9$ and $a_8 = 2187$. Write a rule for the nth term.

$a_n = a_3(r)^{n-3}$

$a_8 = a_3(r)^{8-3}$

$2187 = 9(r)^5$

$243 = r^5$

$r = 3$

$a_3 = 9,\ r = 3$

$\underset{a_1}{1} \xleftarrow{\div 3} \underset{a_2}{3} \xleftarrow{\div 3} \underset{a_3}{9}$

$\boxed{a_n = 1(3)^{n-1}}$

A geometric series is created by adding the numbers of the sequence.

Sigma Notation indicates that a certain number of terms should be added and the formula that describes the sequence.

$$\sum_{i=1}^{n} 5^i$$

last number to plug in (NOT # of terms)

equation that creates the series

first number to plug in

Example 1: $\sum_{i=1}^{4} 5^i = 5^1 + 5^2 + 5^3 + 5^4$

$= 5 + 25 + 125 + 625$

$= 780$

Example 2: $\sum_{i=1}^{6} 3(-2)^i$

$= 3(-2)^1 + 3(-2)^2 + 3(-2)^3 + 3(-2)^4 + 3(-2)^5 + 3(-2)^6$

$= -6 + 12 - 24 + 48 - 96 + 192$

$= 126$

Example 3: $\sum_{i=0}^{5} 48(\frac{1}{2})^i$

$= 48(\frac{1}{2})^0 + 48(\frac{1}{2})^1 + 48(\frac{1}{2})^2 + 48(\frac{1}{2})^3 + 48(\frac{1}{2})^4 + 48(\frac{1}{2})^5$

$= 48 + 24 + 12 + 6 + 3 + 1.5$

$= 94.5$

Example 4: $\sum_{i=3}^{6} (5i - 3)$

$= [5(3)-3] + [5(4)-3] + [5(5)-3] + [5(6)-3]$

$= 12 + 17 + 22 + 27$

$= 78$

Write the following series in sigma notation.

1. 2 + 5 + 8 + 11 + 14

$a_n = dn + a_0$

$a_n = a_1 + d(n-1)$

$a_n = 3n - 1$

$\sum_{i=1}^{5} (3i-1)$

2. 9 + 2 + -5 + -12 + -19 + -26

$\sum_{i=1}^{6} (-7i+16)$

3. 2 + 4 + 8 + 16 + 32 + ...

$\sum_{i=1}^{\infty \text{ or } n} 2(2)^{i-1}$ or $\sum_{i=1}^{\infty} 2^i$