Geometric Series

Standard:

9-12.A.SSE.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

What am I learning today?

How to evaluate a geometric series in sigma notation

How will I show that I learned it?

Expand a series from sigma notation and evaluate it.

Day 2

Finite Geometric Series

Formula for the Sum of a Finite Geometric Series

$$S_n = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

where...

n = # of terms

a_1 = 1st term

r = common ratio

In word problems, if they mention the word "total" or "generation" you are finding the sum!

Example:

1a)   Find the sum of the first 8 terms of the geometric series: 2 - 6 + 18 - 54.. $+162 - 486 + 1458$

$$- 4374$$

$$= -3280$$

$$S_8 = 2\left(\frac{1 - (-3)^8}{1 - (-3)}\right) = -3280$$

1b)   Write the series in sigma notation.

$$\sum_{i=1}^{8} 2(-3)^{i-1}$$

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2)   Find the sum of the geometric series: $\sum_{i=1}^{10} 3(2)^{i-1}$

a) by hand

$$a_1 = 3(2)^{i-1} = 3$$

$$3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 + 768 + 1536$$

$$= 3069$$

b) by using the formula

$$S_{10} = 3\left(\frac{1 - 2^{10}}{1 - 2}\right) = 3069$$

3. Find the sum of the geometric series: $\displaystyle\sum_{i=1}^{7} 2\left(-\frac{3}{2}\right)^{i-1}$

a) by hand

$$a_1 = 2\left(-\frac{3}{2}\right)^{1-1} = 2$$

$$2 - 3 + \frac{9}{2} - \frac{27}{4} + \frac{81}{8} - \frac{243}{16} + \frac{729}{32} = \frac{463}{32}$$

or

$$14.46875$$

b) by using the formula

$$S_7 = 2\left(\frac{1 - \left(-\frac{3}{2}\right)^7}{1 - \left(-\frac{3}{2}\right)}\right) = 2 \cdot \left(\frac{1 + \frac{2187}{128}}{\frac{5}{2}}\right)$$

$$\frac{81}{\cdot 27}$$

$$= 2 \cdot \frac{2}{5}\left(\frac{128}{128} + \frac{2187}{128}\right)$$

$$= \frac{4}{5}\left(\frac{2315 \;\; 463}{128 \;\; 32}\right) = \boxed{\frac{463}{32}}$$

Real World Application: Adapted from <u>Advanced Algebra Georgia</u> (HOLT p. 162)

Real Estate: A 6-year lease states that the annual rent for an office space is $84,000 the first year and will increase by 8% each additional year of the lease. What is the rent in the 6th year? What will the total rent expense be for the 6-year lease?

     n = # of years of the lease
     a_n = amount of rent paid in the nth year
     r = 100% of current rent and 8% increase
         so 100% + 8% or 1.08

     so the sequence rule is $a_n = 84000(1.08)^{n-1}$

     What is the rent in 6 years?

$$a_6 = 84000(1.08)^{6-1}$$

$$= \$123,423.56$$

What will the **TOTAL** rent expense be for the 6-year lease?

Reminder: $S_n = a_1 \left(\dfrac{1 - r^n}{1 - r} \right)$

$$S_6 = 84000 \left(\dfrac{1 - 1.08^6}{1 - 1.08} \right)$$

$$= \$616,218.04$$

Homework:

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\# 28, 29 - 47 odd

You are currently 25 years old. You are going to retire at 65 years old. $n = 40$

A. If you start saving $1000 a year and the interest rate is 5%, how much will you have?

$$S_{40} = 1000\left(\frac{1 - 1.05^{40}}{1 - 1.05}\right) = \$120{,}799.77$$

B. If you save $1000 and the interest rate is 6%, how much will you have?

$$S_{40} = 1000\left(\frac{1 - 1.06^{40}}{1 - 1.06}\right) = \$154761.97$$

C. If you save $3000 and the interest rate is 6%, how much will you have?

$$S_{40} = 3000\left(\frac{1 - 1.06^{40}}{1 - 1.06}\right) = \$464285.90$$