

Warm-up: Solve the following inequalities.

1.  $6(2)^{3x-2} < 54$

2.  $3 \log_3 (x-8) < 12$

## Graphing Logarithms

**MGSE9-12.F.IF.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior

## What am I learning today?

How to graph logarithmic functions

## How will I show that I learned it?

Graph a logarithm and find the characteristics

Reminder: What is the relationship between x and y-values of inverses?

they switch

What is the inverse of a logarithm?

exponential

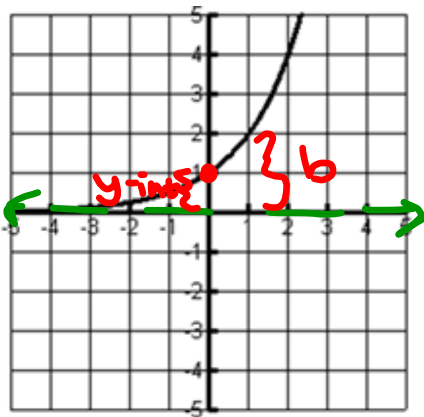
Characteristics of all exponential parent functions:

- a) Horizontal asymptote at  $y=0$
- b) Y-intercept of  $(0,1)$

Characteristics of all logarithmic parent function:

- a) Vertical asymptote at  $x=0$
- b) X-intercept of  $(1,0)$

Exponential



Equation:  $y = 2^x$

D:  $(-\infty, \infty)$  R:  $(0, \infty)$

Max/Min: none Asymp:  $y = 0$

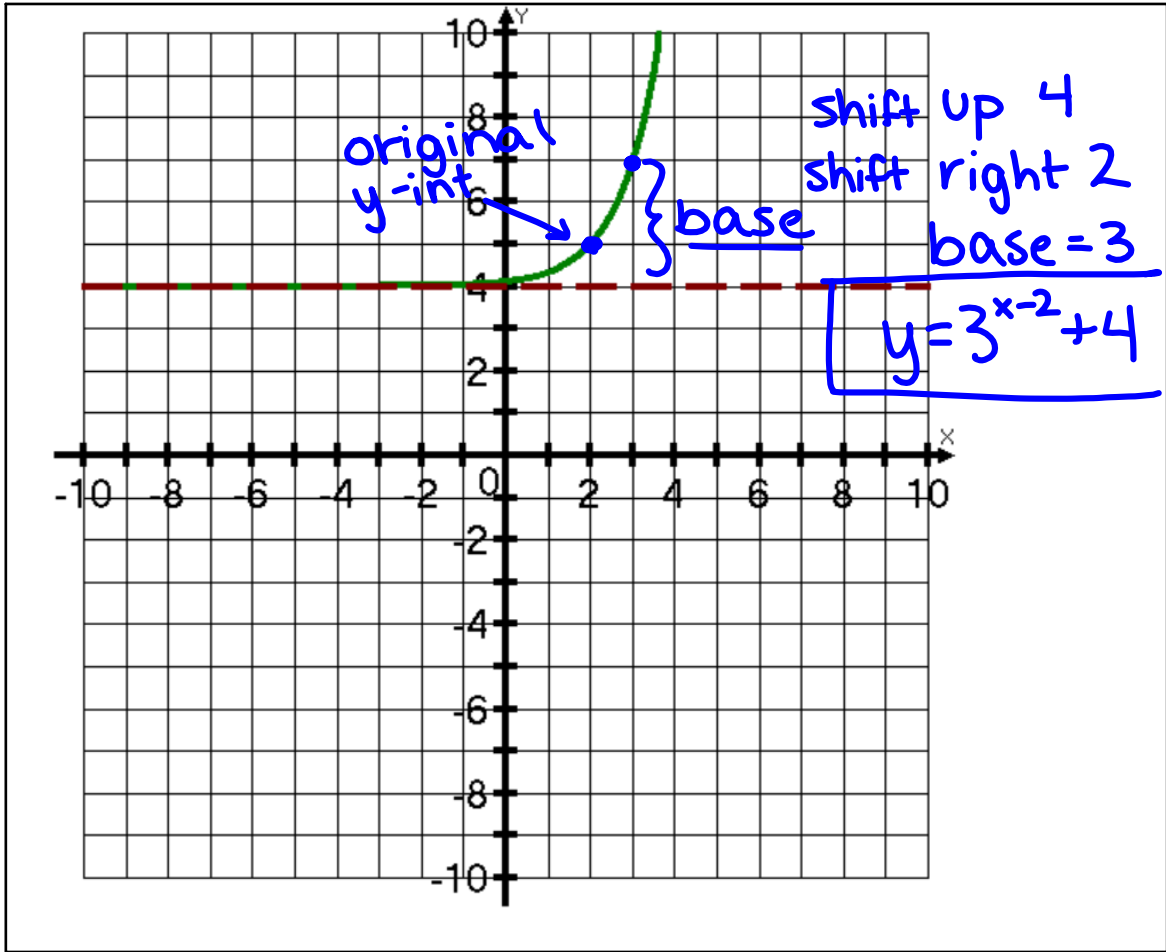
X-Int: none Y-Int:  $(0, 1)$

Inc/Dec/Const:  $(-\infty, \infty)$

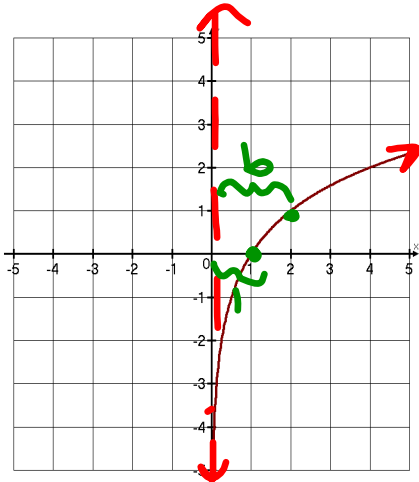
End Behavior:

As  $x \rightarrow -\infty$ ,  $y \rightarrow 0$

As  $x \rightarrow \infty$ ,  $y \rightarrow \infty$



Logarithmic



Equation:  $y = \log_2 x$

D:  $(0, \infty)$  R:  $(-\infty, \infty)$

Max/Min: none Asymp:  $x = 0$

X-Int:  $(1, 0)$  Y-Int: none

Inc/Dec/Const:  $(0, \infty)$

End Behavior:

As  $x \rightarrow 0$ ,  $y \rightarrow -\infty$

As  $x \rightarrow \infty$ ,  $y \rightarrow \infty$

$$f(x) = a \log_B(b(x - h)) + k$$

- reflect over x-axis
- v. dilate
- reflect over y-axis
- h. dilate
- shift horizontally
- shift vertically

Inverse:  $f^{-1}(x) = B^x$

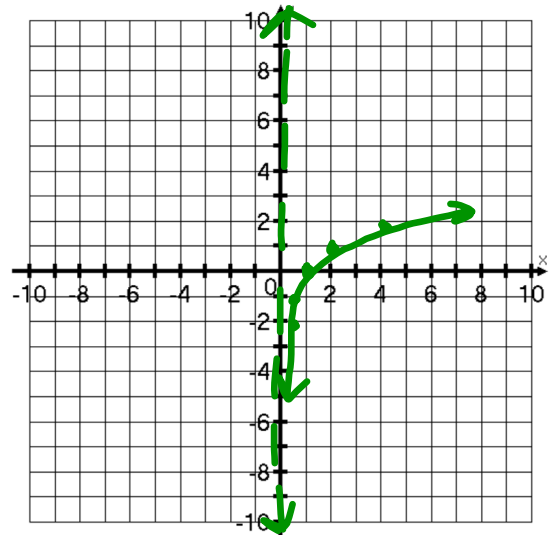
$$f(x) = \log_2 x$$

Exponential Table

x	2 <sup>x</sup>
-2	1/4
-1	1/2
0	1
1	2
2	4

Logarithm Table

x	log <sub>2</sub> x
1/4	-2
1/2	-1
1	0
2	1
4	2



Domain:  $(0, \infty)$       Range:  $(-\infty, \infty)$   
 Asymptote:  $x = 0$       Extrema: none  
 Interval of increase:  $(0, \infty)$   
 X-Intercept:  $(1, 0)$       Y-Intercept: none  
 End Behavior: As  $x \rightarrow 0$ ,  $f(x) \rightarrow -\infty$       As  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$

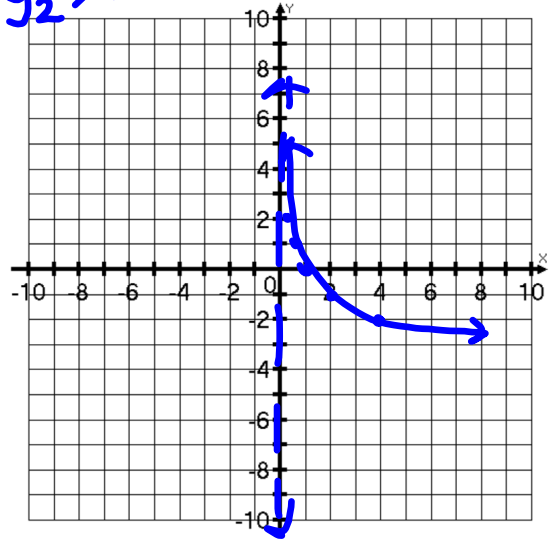
$$f(x) = \log_{\frac{1}{2}} x = -\log_2 x$$

Exponential Table

x	$\frac{1}{2}^x$
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

Logarithm Table

x	$\log_{\frac{1}{2}} x$
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2



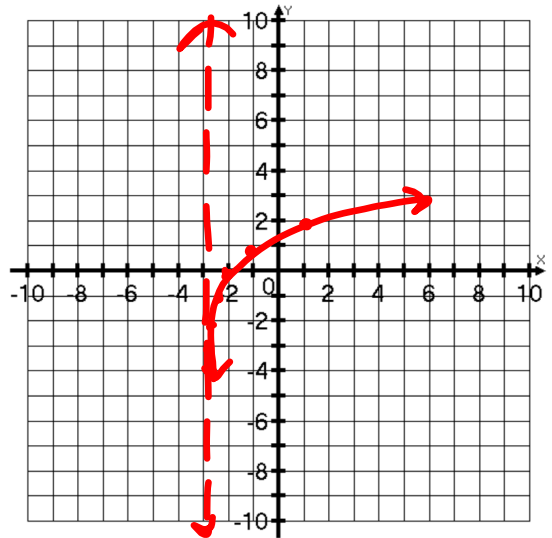
Domain:  $(0, \infty)$  Range:  $(-\infty, \infty)$   
 Asymptote:  $x = 0$  Extrema: none  
 Interval of decrease:  $(0, \infty)$   
 X-Intercept:  $(1, 0)$  Y-Intercept: none  
 End Behavior: As  $x \rightarrow 0$ ,  $f(x) \rightarrow \infty$ . As  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$ .

$$f(x) = \log_2(x + 3)$$

• shift left 3 (x)

Exponential Table      Logarithm Table      Transformed Table

x	$2^x$	x	$\log_2 x$	x-3	y
-2	$\frac{1}{4}$	$\frac{1}{4}$	-2	$-\frac{11}{4}$	-2
-1	$\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{5}{2}$	-1
0	1	1	0	-2	0
1	2	2	1	-1	1
2	4	4	2	1	2



Y-int  
 $\log_2(0+3)$   
 $\log_2 3$

Domain:  $(-3, \infty)$  Range:  $(-\infty, \infty)$   
 Asymptote:  $x = -3$  Extrema: none  
 Interval of increase:  $(-3, \infty)$   
 X-Intercept:  $(-2, 0)$  Y-Intercept:  $(0, \log_2 3)$   
 End Behavior: As  $x \rightarrow -3$ ,  $f(x) \rightarrow -\infty$ . As  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$ .

$f(x) = \log_2(x + 2) - 3$   
 • shift down 3 (y) and left 2 (x)

Exponential Table      Logarithm Table      Transformed Table

$x$	$2^x$	$x$	$\log_2 x$	$x-2$	$y-3$
-2	1/4	1/4	-2	-7/4	-5
-1	1/2	1/2	-1	-3/2	-4
0	1	1	0	-1	-3
1	2	2	1	0	-2
2	4	4	2	2	-1

x-int  
 $0 = \log_2(x+2) - 3$   
 $3 = \log_2(x+2)$   
 $2^3 = x+2$   
 $8 = x+2 \therefore x=6$

Domain:	$(-2, \infty)$	Range:	$(-\infty, \infty)$
Asymptote:	$x = -2$	Extrema:	none
Interval of increase:	$(-2, \infty)$		
X-Intercept:	$(6, 0)$	Y-Intercept:	$(0, -2)$
End Behavior:	As $x \rightarrow -2$ , $f(x) \rightarrow -\infty$ .	As $x \rightarrow \infty$ , $f(x) \rightarrow \infty$ .	

• reflect over x-axis (y)  
 $f(x) = -\log_5(x + 4)$   
 • shift left 4 (x)

Exponential Table      Logarithm Table      Transformed Table

$x$	$\frac{1}{2}^x$	$x$	$\log_5 x$	$x-4$	$-y$
-2	4	4	-2	0	2
-1	2	2	-1	-2	1
0	1	1	0	-3	0
1	1/2	1/2	1	-7/2	-1
2	1/4	1/4	2	-11/2	-2

Domain:	$(-4, \infty)$	Range:	$(-\infty, \infty)$
Asymptote:	$x = -4$	Extrema:	none
Interval of increase:	$(-4, \infty)$		
X-Intercept:	$(-3, 0)$	Y-Intercept:	$(0, 2)$
End Behavior:	As $x \rightarrow -4$ , $f(x) \rightarrow -\infty$ .	As $x \rightarrow \infty$ , $f(x) \rightarrow \infty$ .	

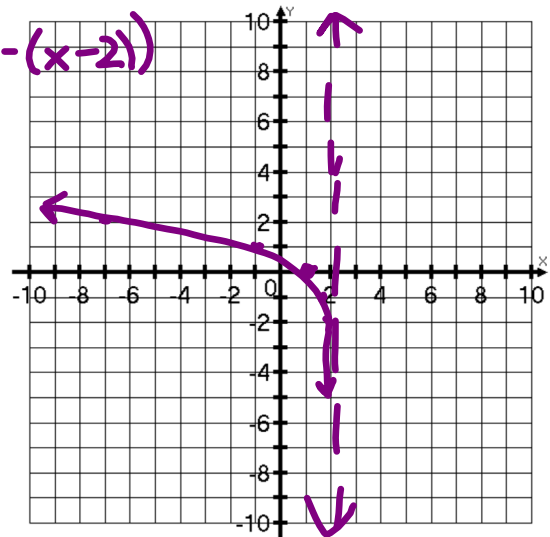
· reflect over y-axis (x)

$$f(x) = \log_3(-x + 2) = \log_3(-(x-2))$$

· shift right 2 (x)

Exponential Table      Logarithm Table      Transformed Table

x	$3^x$	x	$\log_3 x$	-x+2	y
-2	1/9	1/9	-2	17/9	-2
-1	1/3	1/3	-1	5/3	-1
0	1	1	0	1	0
1	3	3	1	-1	1
2	9	9	2	-7	2

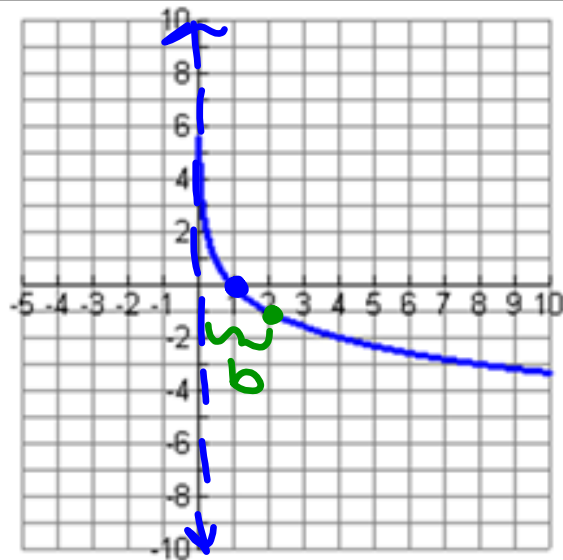


Domain:  $(-\infty, 2)$       Range:  $(-\infty, \infty)$   
 Asymptote:  $x=2$       Extrema: none  
 Interval of decrease:  $(-\infty, 2)$   
 X-Intercept:  $(1, 0)$       Y-Intercept:  $(0, \log_3 2)$   
 End Behavior: As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow \infty$ . As  $x \rightarrow 2$ ,  $f(x) \rightarrow -\infty$ .

Ex. A

Equation:

$$y = -\log_2 x$$

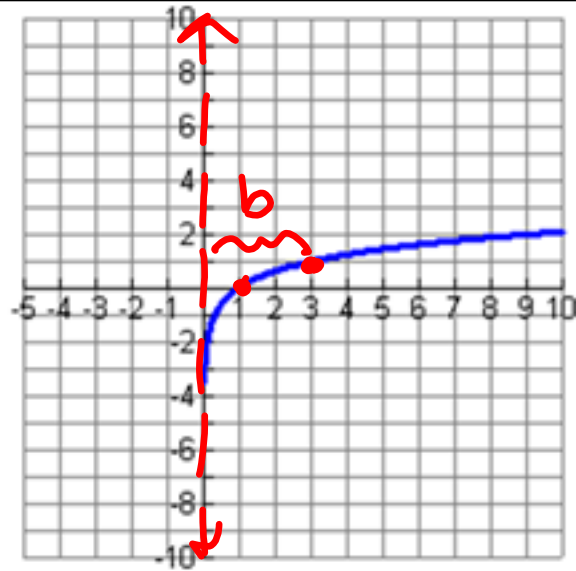




Ex. B

Equation:

$$y = \log_3 x$$

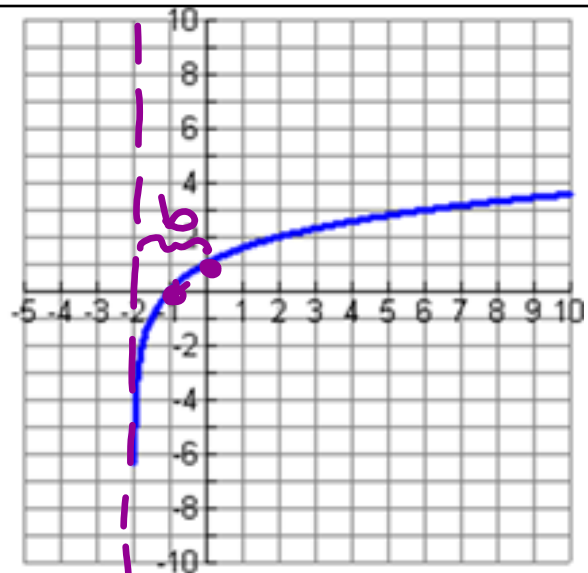


Ex. C

Equation:

· shift left 2

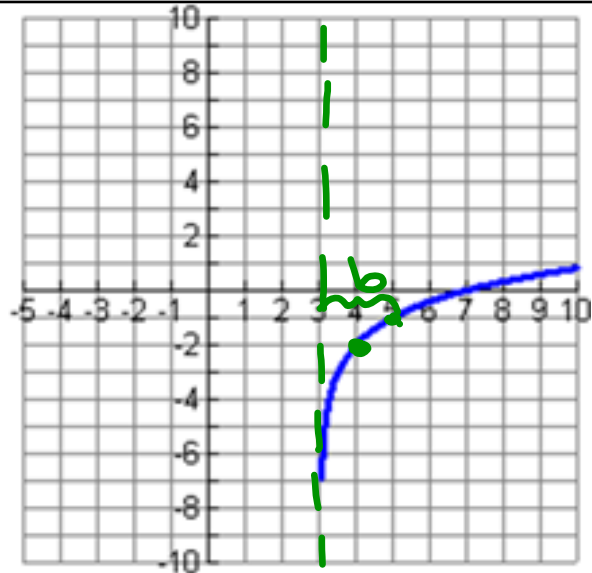
$$y = \log_2(x+2)$$



Ex. D

Equation:

shift right 3  
 shift down 2  
 $y = \log_2(x-3) - 2$



Ex. E

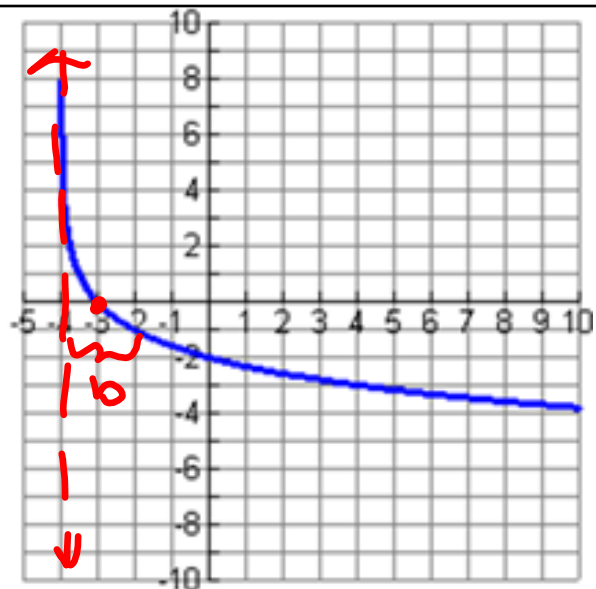
Equation:

- reflect over x-axis
- shift left 4

$$y = -\log_2(x+4)$$

or

$$y = \log_{1/2}(x+4)$$



Ex. F

Equation:

$$y = \log_2(-x)$$

