Intro to Exponential Functions

F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

MGSE9-12.F.IF.7e Graph exponential functions, showing intercepts and end behavior.

Feb 28-10:54 AM

What am I learning today?

How to solve and describe the characteristics of an exponential function

How will I show that I learned it?

Solve, graph and identify the characteristics of exponential graphs

Aug 18-1:21 PM

Vocabulary:

Exponential Function - a function with a positive constant base and a variable exponent. Example: $f(x) = 2^x$

Asymptote - A value that a function approaches but never reaches. This is represented graphically by a dotted line.

Exponential Growth - An exponential graph whose magnitude of rate of change is increasing from left to right. Caused by a base greater than 1. Example: $f(x) = 2^x$

Exponential Decay - An exponential graph whose magnitude of rate of change is decreasing from left to right. Caused by a base greater than 0 but less than 1. Example: $f(x) = (1/2)^x$

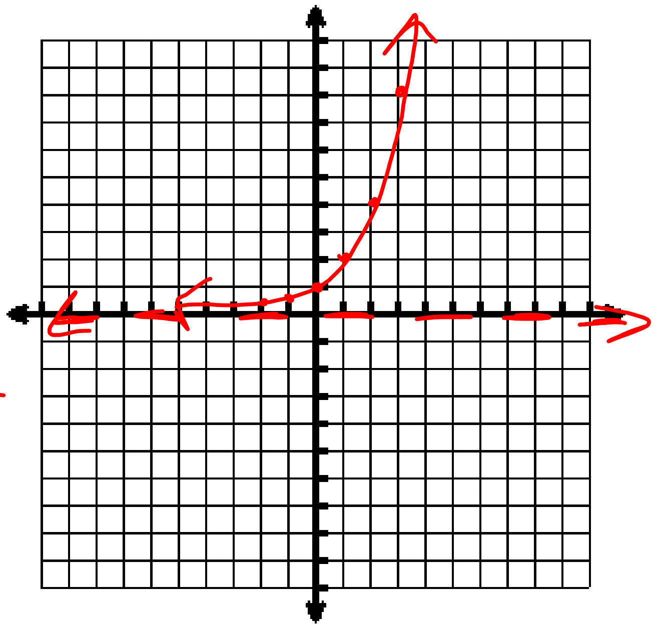
Feb 28-10:55 AM

Parent Function

Exponential Functions ($y = b^x$) when graphed are flat on one side and steep on the other.

$y = 2^x$

x	y
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$



Nov 13-1:30 PM

Basics about Exponential Graphs

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptote: $y = 0$

Extrema: none

b $>$ 1

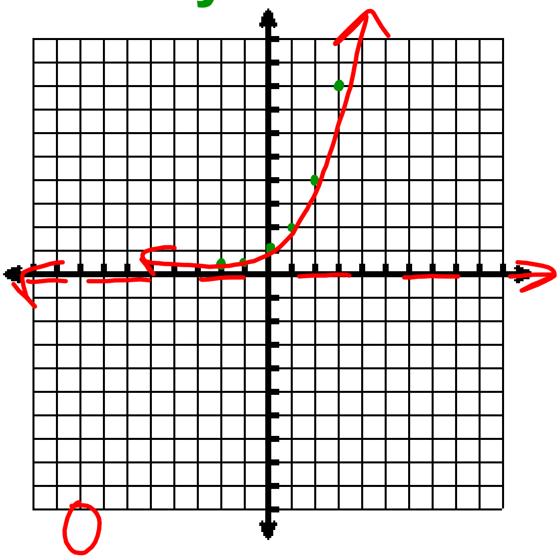
Exponential Growth

x-int: none y-int: $(0, 1)$

End Behavior: As x $\to -\infty$, y $\to 0$

As x $\to \infty$, y $\to \infty$

$y = 2^x$



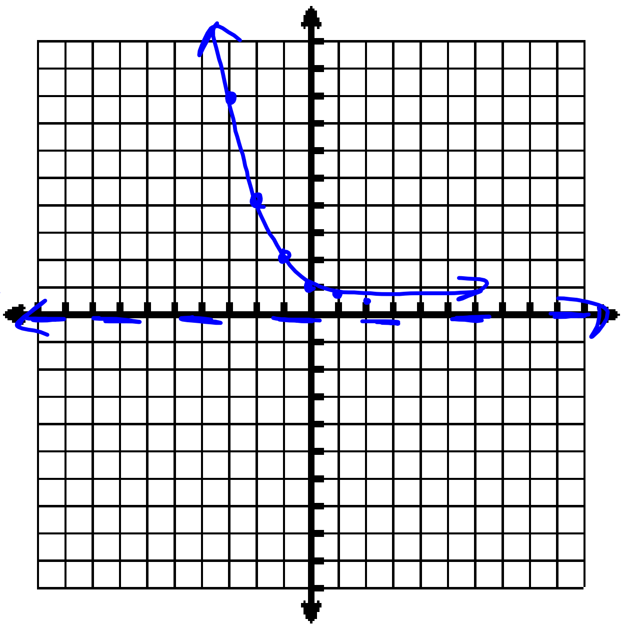
Nov 13-1:30 PM

Parent Function

Exponential Functions ($y = b^x$) when graphed are flat on one side and steep on the other.

$y = 1/2^x$

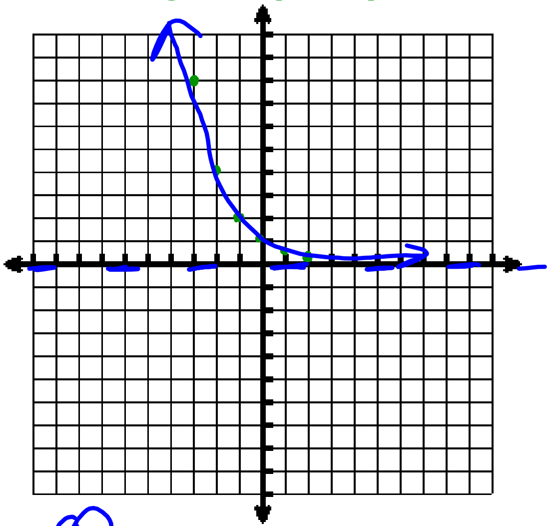
x	y
-2	$\frac{1}{2}^{-2} = 4$
-1	$\frac{1}{2}^{-1} = 2$
0	$\frac{1}{2}^{0} = 1$
1	$\frac{1}{2}^{1} = \frac{1}{2}$
2	$\frac{1}{2}^{2} = \frac{1}{4}$



Nov 13-1:30 PM

Basics about Exponential Graphs

$y = (1/2)^x$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptote: $y = 0$

Extrema: none

b $\leq$ 1

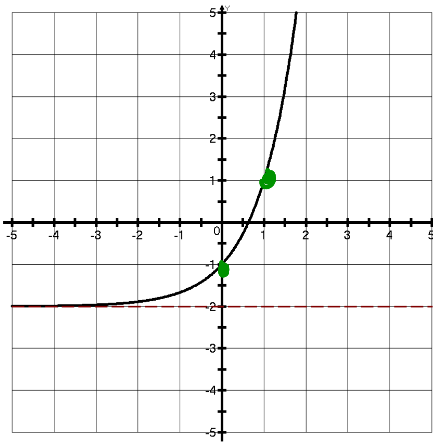
Exponential Decay

x-int: none y-int: $(0, 1)$

End Behavior: As x $-\infty$, y ∞

As x ∞, y 0

Nov 13-1:30 PM



Domain: $(-\infty, \infty)$ Range: $(-2, \infty)$

Asymptote: $y = -2$ Extrema: none

b $>$ 1 Exponential Growth

X-Intercept: $(.6, 0)$ Y-Intercept: $(0, -1)$

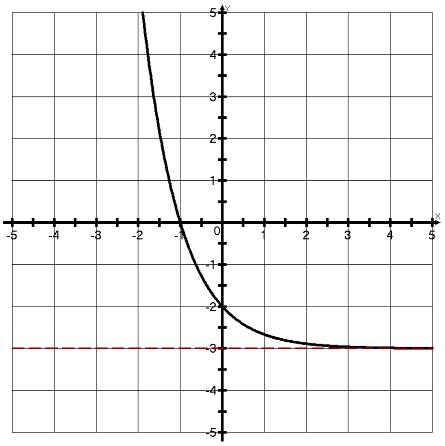
End Behavior: As x $\to -\infty$, y $\to -2$, As x $\to \infty$, y $\to \infty$

Rate of change on the interval $0 \le x \le 1$: 2

$(0, -1)\ (1, 1)$ $[0, 1]$

$$\frac{1-(-1)}{1-0}$$

Oct 23-12:57 PM



Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

Asymptote: $y = -3$

Extrema: none

b $<$ 1

Exponential Decay

X-Intercept: $(-1, 0)$

Y-Intercept: $(0, -2)$

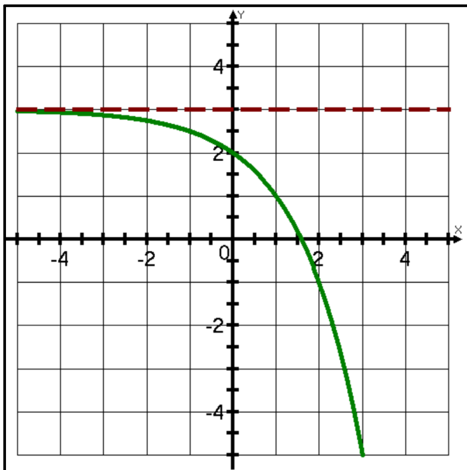
End Behavior: As $x \to -\infty$, $y \to \infty$; As $x \to \infty$, $y \to -3$

Rate of change on the interval $-1 \leq x \leq 0$: -2

$(-1, 0)\ (0, -2)$

$$\frac{-2-0}{0-(-1)}$$

Oct 23-12:57 PM



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

Asymptote: $y = 3$

Extrema: none

b $>$ 1

Exponential Growth

X-Intercept: $(1.6, 0)$

Y-Intercept: $(0, 2)$

End Behavior: As $x \to -\infty$, $y \to 3$; As $x \to \infty$, $y \to -\infty$

Rate of change on the interval $0 \leq x \leq 1$: -1

$(0, 2)\ (1, 1)$

$$\frac{1-2}{1-0}$$

Oct 23-12:57 PM

Product Rule

$a^x \cdot a^y = a^{x+y}$

$x^3 \cdot x^4 = x^{3+4} = x^7$

$2^x \cdot 2^3 = 2^{x+3}$

Power Rule

$(a^x)^y = a^{x \cdot y}$

$(x^3)^4 = x^{3 \cdot 4} = x^{12}$

$(2^3)^{x+2} = 2^{3x+6}$ *

Quotient Rule

$\frac{a^x}{a^y} = a^{x-y}$

$\frac{x^5}{x^3} = x^{5-3} = x^2$

$\frac{3^x}{3^2} = 3^{x-2}$

Negative Exponents

$a^{-x} = \frac{1}{a^x}$

$x^{-3} = \frac{1}{x^3}$ $\frac{1}{x^{-3}} = x^3$

$\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$

Oct 28-1:55 PM

Steps for Solving an Exponential:

1. Isolate the base.
2. Write both sides of the equation as exponential expressions with LIKE bases.
3. Set the exponents equal to each other.
4. Distribute or multiply exponents on same side if raising a power to a power.
5. Solve for the unknown.

Jan 22-1:44 PM

$$4^{x-3} + 2 = 66$$
$$\underline{\quad -2 \quad -2}$$
$$4^{x-3} = 64$$
$$(2^2)^{x-3} = (2^6)$$
$$2(x-3) = 6$$
$$2x - 6 = 6$$
$$+6 \quad +6$$
$$2x = 12$$
$$\underline{\div 2 \quad \div 2}$$
$$x = 6$$

Jan 22-1:47 PM

We use exponent rules to change bases to be the same. Then we use the exponent property of equality to solve.

Ex. $2^{2x} = 16$

$$\cancel{2}^{2x} = \cancel{2}^{4}$$
$$2x = 4$$
$$\boxed{x = 2}$$

Apr 1-6:16 PM

Ex. 2 $9^{3x} = 27^{1-x}$

$(\cancel{3}^2)^{3x} = (\cancel{3}^3)^{1-x}$

$2(3x) = 3(1-x)$

$6x = 3 - 3x$

$9x = 3$

$\boxed{x = \frac{1}{3}}$

Ex. 3 $4^{4x-1} > (1/32)^{x+2}$

$(2^2)^{4x-1} > \left(\frac{1}{2^5}\right)^{x+2}$

$(\cancel{2}^2)^{4x-1} > (\cancel{2}^{-5})^{x+2}$

$2(4x-1) > -5(x+2)$

$8x - 2 > -5x - 10$

$13x > -8$

$x > -\frac{8}{13}$

$\boxed{\left(-\frac{8}{13}, \infty\right)}$

Apr 1-6:17 PM

Ex. 4 $9^{x-3} + 2 \leq 29$

$9^{x-3} \leq 27$

$(\cancel{3}^2)^{x-3} \leq \cancel{3}^3$

$2(x-3) \leq 3$

$2x - 6 \leq 3$

$2x \leq 9$

$x \leq \frac{9}{2}$

$\boxed{\left(-\infty, \frac{9}{2}\right]}$

Ex. 5 $49^{x+2} = 343^x$

$(\cancel{7}^2)^{x+2} = (\cancel{7}^3)^x$

$2(x+2) = 3(x)$

$2x + 4 = 3x$

$\boxed{4 = x}$

Apr 1-6:19 PM

Ex. 6

$$8^{x-3} = \left(\frac{1}{4}\right)^{2x}$$

$$\left(2^3\right)^{x-3} = \left(\frac{1}{2^2}\right)^{2x}$$

$$\left(\cancel{2}^3\right)^{x-3} = \left(\cancel{2}^{-2}\right)^{2x}$$

$$3(x-3) = -2(2x)$$

$$3x-9 = -4x$$

$$-9 = -7x$$

$$\boxed{x = \frac{9}{7}}$$

Jan 22-2:48 PM

Apr 26-1:06 PM