Application Problems

1. Normal Exponential Growth Problems

$$P(t) = P_0 e^{kt}$$

P(t) = amount at time *t*

P_0 = intial amount

k = growth rate

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Ex. 1) The population of a city grows according to the normal exponential growth model. The population of a city was 257,326 in 2010, and in 2015 the population had grown to 258,877.

A. Find the growth rate.

$P = P_0 e^{kt}$

$258877 = 257326 e^{5k}$

$1.006 = e^{5k}$

$\frac{\ln 1.006}{5} = \frac{5k}{5}$

$.0012 = k$

B. Predict the population in 2020.

$P = 257326 e^{10(.0012)} = 260{,}437$

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Ex. 2) The number of bacteria in a culture is given by the normal growth model. The initial population is 100 bacteria and after 5 hours the population grows to 300 bacteria.

A. Find the growth rate.

$300 = 100e^{5k}$

$3 = e^{5k}$

$\ln 3 = 5k$

$k = \frac{\ln 3}{5} = .22$

B. Estimate the time required for the population to double in size.

$200 = 100e^{.22t}$

$2 = e^{.22t}$

$\ln 2 = .22t$

$t = \frac{\ln 2}{.22} = 3.15$ hours

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Application Problems

2. Compound Interest Problems

Periodically Compounded Interest	Continuously Compounded Interest
$A = P\left(1+\frac{r}{n}\right)^{nt}$	$A = Pe^{rt}$

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Ex. 3) An amount of $1500 is deposited into a bank account paying an annual interest of 4.3%, compounded quarterly.

A. Find the balance in the account after 6 years.

$A = 1500\left(1 + \frac{.043}{4}\right)^{(4)(6)} = \$1938.84$

B. How many years would it take the account to double in size?

$2 = \left(1 + \frac{.043}{4}\right)^{4t}$

$2 = 1.01075^{4t}$

$\frac{\log_{1.01075} 2}{4} = \frac{4t}{4}$

$t = 16.21$ years

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Ex. 4) $3200 is deposited into an account that is compounded <u>continuously.</u>

A. If it makes 7.2% interest, how much will it be worth in 5 years?

$A = 3200e^{.072 \cdot 5} = \$4586.65$

B. What interest rate is required to make it worth twice as much in 10 years?

$6400 = 3200e^{10r}$

$2 = e^{10r}$

$\frac{\ln 2}{10} = \frac{10r}{10}$

$r = .069 \approx 6.9\%$

C. How many years will it take for the account to be worth $10,000 at an 8% interest rate?

$10000 = 3200e^{.08t}$

$3.125 = e^{.08t}$

$\ln 3.125 = .08t$

$t = \frac{\ln 3.125}{.08}$

$t = 14.24$ years

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Application Problems

3. Normal Exponential Decay Problems

$$P(t) = P_0 e^{-kt}$$

P(t) = amount at time *t*

P_0 = intial amount

k = growth rate

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Depreciation:

$$A = P(1 - r)^t$$

Half-life:

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{n}}$$

n = half-life

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Ex. 5) A bone that originally contained 150 mg of carbon–14 now contains 85 mg of that isotope. Determine the age of the bone to the nearest 100 years if the half life of carbon-14 is 5570 years.

$A = A_0\left(\frac{1}{2}\right)^{t/n}$

$85 = 150\left(\frac{1}{2}\right)^{t/5570}$

$\frac{17}{30} = \left(\frac{1}{2}\right)^{t/5570}$

$\log_{\frac{1}{2}}\left(\frac{17}{30}\right) = \frac{t}{5570}$

$t = 5570\log_{\frac{1}{2}}\left(\frac{17}{30}\right)$

$t = 4564.21$

$t \approx 4600$ years

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Ex. 6) You bought a brand new car for $32,000 yesterday. The depreciation rate for a new car is approximately 20% of its value.

A. If you plan on selling it when you graduate from college (in 8 years) what would be its value?

$A = 32000(1-.2)^8 = \$5368.71$

B. If you plan to sell it when it reaches a value of $20,000, how many years will you have it?

$20000 = 32000(.8)^x$

$.625 = .8^x$

$\log_{.8}.625 = x$

$x = 2.11$ years

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Ex. 7) If there are 200 milligrams of carbon14 (halflife of 5570 years) in a sample, how many years will it take for the sample to have...

100 mg? $\frac{100}{200} = \frac{1}{2}$ 5570 years (1 half life)

50 mg? $\frac{50}{200} = \frac{1}{4} = \frac{1}{2}^{2}$ 11,140 yrs (2 half-lives)

25 mg? $\frac{25}{200} = \frac{1}{8} = \frac{1}{2}^{3}$ 16710 years (3 half lives)

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