Properties of Logarithms

EQ: What are the properties of logarithms?

Standards:

MCC9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *(Limit to exponential and logarithmic functions.)*

MCC9-12.A.SSE.3c Use the properties of exponents to transform expressions for exponential functions.

MCC9-12.F.IF8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *(Limit to exponential and logarithmic functions.)*

Product Property: $\log_x ab = \log_x a + \log_x b$

Ex. 1 $\log_a 4x = \log_a 4 + \log_a x$

1

Ex. 2 $\log_3 6 = \log_3 (2 \cdot 3)$

$= \log_3 2 + \log_3 3$

$= \log_3 2 + 1$

Properties of Logarithms

Product Property: $\log_b u + \log_b v = \log_b uv$

Examples:

1. Condense: $\log_2 3 + \log_2 4 + \log_2 k$

$= \log_2(3 \cdot 4 \cdot k) = \log_2 12k$

2. Expand: $\log_{11} 21xy = \log_{11}(3 \cdot 7 \cdot x \cdot y)$

$= \log_{11} 3 + \log_{11} 7 + \log_{11} x + \log_{11} y$

Quotient Property: $\log_x (a/b) = \log_x a - \log_x b$

Ex. 1 $\log_a x/y = \log_a x - \log_a y$

Ex. 2 $\log_3 1/3 = \log_3 1 - \log_3 3$

$= 0 - 1 = -1$

Quotient Property: $\log_b u - \log_b v = \log_b \frac{u}{v}$

Examples:

1. Condense: $\log_3 5 - \log_3 20 - \log_3 y$

$= \log_3\left(\frac{5}{20 \cdot y}\right) = \log_3\left(\frac{1}{4y}\right)$

2. Expand: $\ln\frac{(x+3)}{6} = \ln\frac{(x+3)}{2 \cdot 3}$

$= \ln(x+3) - \ln 2 - \ln 3$

Power Property: $\log_x a^b = b(\log_x a)$

Ex. 1 $\log_a 3^5 = 5\log_a 3$

Ex. 2 $\log_2 4^3 = 3(\log_2 4) = 3(2) = 6$

$\log_2 (2^2)^3 = \cancel{\log_2 2^6} = 6$

Property of Powers: $\log_b x^y = y\log_b x$

Examples:

1. Condense: $3\log_4 x = \log_4 x^3$

2. Condense: $4\log_7 2 = \log_7 2^4 = \log_7 16$

3. Expand: $\log_9 x^5 = 5\log_9 x$

4. Expand: $\log_3 144 = \log_3(2^4 \cdot 3^2)$
$= \log_3 2^4 + \log_3 3^2$
$= 4\log_3 2 + 2$

5. Expand: $\log_2 32x^7 = \log_2(2^5 \cdot x^7)$
$= \log_2 2^5 + \log_2 x^7$
$= 5 + 7\log_2 x$

Recommended Steps

EXPAND:

- convert any perfect power # to its smallest base.
- convert any radicals to a power
- deal with expanding top (+) & bottom (-) at same time
- finish with rolling exponents to front of each term
- "clean up" any $\log_b b^\#$ situations

Condense:

- Convert any # term back to $\log_b b^\#$
- reorganize so all (+) terms and (-) are together
- roll exponents to back of each term
- deal with placement of (+) terms (top) & (-) terms (bottom) at same time

$$\frac{7x^3}{y} = \log_2 7 + \log_2 x^3 - \log_2 y$$
$$= \log_2 7 + 3\log_2 x - \log_2 y$$

2. $\log_5 2x^6\sqrt{y} = \log_5(2 \cdot x^6 \cdot y^{1/2})$
$$= \log_5 2 + \log_5 x^6 + \log_5 y^{1/2}$$
$$= \log_5 2 + 6\log_5 x + \tfrac{1}{2}\log_5 y$$

3. $\log_7 \dfrac{49y}{3x^2} = \log_7\left(\dfrac{7^2 \cdot y}{3 \cdot x^2}\right)$
$$= \log_7 7^2 + \log_7 y - \log_7 3 - \log_7 x^2$$
$$= 2 + \log_7 y - \log_7 3 - 2\log_7 x$$

4. $\ln \dfrac{3y^4}{wx^3z^2} = \ln 3 + \ln y^4 - \ln w - \ln x^3 - \ln z^2$
$$= \ln 3 + 4\ln y - \ln w - 3\ln x - 2\ln z$$

Condense:

1. log6 + 3log2 - log3

$= \log 6 + \log 2^3 - \log 3$

$= \log\left(\frac{6 \cdot 2^3}{3}\right) = \log 16$

2. $4\log_6 5 + 3\log_6 x + \log_6 y + 2\log_6 z$

$= \log_6 5^4 + \log_6 x^3 + \log_6 y + \log_6 z^2$

$= \log_6(5^4 \cdot x^3 y z^2) = \log_6(625x^3yz^2)$

3. ln 13 - ln 7 + ln 2 - ln 6

$= \ln\left(\frac{13 \cdot \cancel{2}^{1}}{7 \cdot \cancel{6}_{3}}\right) = \ln\left(\frac{26}{42}\right) = \ln\left(\frac{13}{21}\right)$

4. $2 + \frac{1}{4}\log_8 x - \log_8 5 - 3\log_8 y$

$= 2\log_8 8 + \frac{1}{4}\log_8 x - \log_8 5 - 3\log_8 y$

$= \log_8 8^2 + \log_8 x^{1/4} - \log_8 5 - \log_8 y^3$

$= \log_8\left(\frac{8^2 \cdot x^{1/4}}{5 \cdot y^3}\right) = \log_8\left(\frac{64x^{1/4}}{5y^3}\right)$

5. $\log_2 x + \log_2(x+1)$

$= \log_2[x(x+1)] = \log_2(x^2+x)$

Solve:

1. $\log_3 7 = \dfrac{\log 7}{\log 3} = \dfrac{\ln 7}{\ln 3}$

$= 1.77$

Change-of-Base Formula: $\log_b t = \dfrac{\log t}{\log b}$

Evaluate:

1. $\log_3 7$

$\log_2 8 = \dfrac{\log 8}{\log 2} = 3$

2. $\log_6 24$

$= \dfrac{\ln 24}{\ln 6}$

$= 1.77$

3. $\log_9 ½$

4. $\log_{¼} 8$

Use properties of logarithms to evaluate

$\log_c 2 = 1.44$ $\log_c 3 = 1.75$

1) $\log_c 8 = \log_c 2^3$

$= 3(\log_c 2)$

$= 3(1.44)$

$= 4.32$

3) $\log_c .5 = \log_c(\frac{1}{2})$

$= \log_c 2^{-1} = -1 \log_c 2 = -1.44$

$= \log_c 1 - \log_c 2$

$= 0 - 1.44$

2) $\log_c 18 = \log_c(2 \cdot 3^2)$

$= \log_c 2 + \log_c 3^2$

$= \log_c 2 + 2\log_c 3$

$= (1.44) + 2(1.75)$

$= 1.44 + 3.5 = 4.94$

4) $\log_c .75 = \log_c\left(\frac{3}{2^2}\right)$

$= \log_c 3 - \log_c 2^2$

$= \log_c 3 - 2\log_c 2$

$= (1.75) - 2(1.44)$

$= 1.75 - 2.88 = -1.13$

Attachments

Brody_Roll Like a Log.xspf

Brody_Roll Like a Log 2.xspf