Solving Logarithmic and Exponential Equations

MGSE9-12.F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

MGSE9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MGSE9-12.F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.

What am I learning today?

How to use logarithmic properties to solve equations

How will I show that I learned it?

Solve equations using exponential and logarithmic operations and properties

Properties of the Natural Logarithm

$\ln e = 1$

$\ln 1 = 0$

$e^{\ln x} = x$

Reminder: Change of base formula

$$\log_a b = \frac{\log b}{\log a} = \frac{\ln b}{\ln a}$$

Example: $\log_4 12 = \frac{\log 12}{\log 4} = 1.79$

$4^{1.79} \overset{\checkmark}{=} 12$

Solving Exponential Equations

Type 1: Powers of the same base

- Isolate the exponential part(s).
- Eliminate the base leaving only the exponential part.
- Always check your answers!!

1) $4^{3x} = 8^{x+1}$

$(2^2)^{3x} = (2^3)^{x+1}$

$2(3x) = 3(x+1)$

$6x = 3x + 3$

$3x = 3$

$\boxed{x = 1}$

2) $\left(\frac{1}{25}\right)^{2x+3} = 5^{x-5}$

$(5^{-2})^{2x+3} = 5^{x-5}$

$-2(2x+3) = x - 5$

$-4x - 6 = x - 5$

$-6 = 5x - 5$

$-1 = 5x$

$\boxed{x = -\frac{1}{5}}$

Type 2: Bases are NOT powers of the same base

- Isolate the exponential part(s).
- Apply a logarithm to each side. Use change of base to help solve. to check your work.
- Always check your answers!!

Examples: Solve for x.

1) $2^x = 7$

$\log_2 2^x = \log_2 7$

$\boxed{x = \log_2 7}$

2) $40e^{0.6x} + 20 = 240$

$$-20 \quad -20$$

$$\frac{40e^{0.6x}}{40} = \frac{220}{40}$$

$$e^{0.6x} = \frac{11}{2}$$

$$\cancel{\ln e}^{0.6x} = \ln\left(\frac{11}{2}\right)$$

$$0.6x = \ln\left(\frac{11}{2}\right)$$

$$x = \frac{\ln\left(\frac{11}{2}\right)}{0.6} \cdot \frac{10}{10}$$

$$= \frac{10\ln\left(\frac{11}{2}\right)}{6} = \frac{5\ln\left(\frac{11}{2}\right)}{3}$$

3) $2^x = 3^{x-1}$

(start on side with more difficult exponent)

$$\log_3 2^x = \log_3 3^{x-1}$$

$$x \cdot \log_3 2 = x - 1$$

$$-x \quad -x$$

$$x \cdot \log_3 2 - x = -1$$

$$x(\log_3 2 - 1) = -1$$

$$\boxed{x = \frac{-1}{\log_3 2 - 1}}$$

Solving Logarithmic Equations

Type 3: Logs on each side have same base.

- Every term has $\log_a(?)$. Condense until each side has only one $\log_a(?)$.

- Set the answers equal to each other. Then solve.

* Always check your answers!! There are extraneous situations ($\log_b x > 0$)

Examples: Solve for x.

1) $\log_7(5x - 1) = \log_7(x + 7)$

$5x - 1 = x + 7$

$4x - 1 = 7$

$4x = 8$

$\boxed{x = 2}$?

$5(2) - 1$ pos? ✓

$2 + 7$ pos? ✓

2) $\ln x - \ln 9 = \ln 3$

$$\ln \frac{x}{9} = \ln 3$$

$$\frac{x}{9} = 3$$

$$\boxed{x = 27}$$

Type 4: Log on only one side of the equation!

- Get $\text{Log}_x(?)$ by itself on the one side. Condense if necessary.

- Change the log equation to exponent form. Then solve.

- Always check your answers!! There are extraneous situations. ($\log_b x > 0$)

Examples: Solve for x.

1) $\log_5(3x + 1) = 2$

$$5^{\log_5(3x+1)} = 5^2$$

$$3x+1=25$$

$$3x=24$$

$$\boxed{x=8}$$

$3(8)+1$ ✓ pos?

2) $\log_8(x - 3) + \log_8(x + 4) = 1$

$$\log_8[(x-3)(x+4)] = 1$$

$$\log_8(x^2+x-12) = 1$$

$$\cancel{8^{\log_8}}(x^2+x-12) = 8^1$$

$$x^2+x-12=8$$

$$x^2+x-20=0$$

$$(x+5)(x-4)=0$$

$\cancel{x=-5}$ $\boxed{x=4}$

pos? pos?

Mixed Examples:

1) $9(12^{3x-5}) - 75 = 1221$

$9(12^{3x-5}) = 1296$

$12^{3x-5} = 144$

$12^{3x-5} = 12^2$

$3x - 5 = 2$

$3x = 7$

$\boxed{x = \frac{7}{3}}$

2) $\log_4 x + \log_4(x^2 - 3) = \log_4 3x$

$\log_4[x(x^2-3)] = \log_4 3x$

$x(x^2-3) = 3x$

$x^3 - 3x = 3x$

$x^3 - 6x = 0$

$x(x^2-6) = 0$

$\cancel{x = 0}$

$x^2 - 6 = 0$

$x^2 = 6$

$\boxed{x = \sqrt{6}}$ $\cancel{x = -\sqrt{6}}$

3) $\ln x + \ln(x - 5) = 1$

$\ln[x(x-5)] = 1$

$\ln[x^2 - 5x] = 1$

$x^2 - 5x = e^1$

$x^2 - 5x - e = 0$

$(-5)^2 - 4(1)(-e) = 25 + 4e$

$\boxed{x = \frac{5 + \sqrt{25+4e}}{2}}$

$\cancel{x = \frac{5 - \sqrt{25+4e}}{2}}$

4) $16^{2x+1} = \frac{1}{32}$

$(2^4)^{2x+1} = 2^{-5}$

$4(2x+1) = -5$

$8x + 4 = -5$

$8x = -9$

$x = -\frac{9}{8}$

5) $15 - 2\log_4(28x + 8) = 3$

$-2\log_4(28x+8) = -12$

$\log_4(28x+8) = 6$

$28x+8 = 4^6$

$28x+8 = 4096$

$28x = 4088$

$\boxed{x = 146}$

pos? ✓

6) $5^{x-3} = -341$

no solution

7) $\log_5(9x - 1) = \log_5(4x - 16)$

$9x-1 = 4x-16$

$5x-1 = -16$

$5x = -15$

$\cancel{x = -3}$ pos?

no solution

8) $3\ln(-x) + 9 = 20$

$3\ln(-x) = 11$

$\ln(-x) = \frac{11}{3}$

$-x = e^{11/3}$

$\boxed{x = -e^{11/3}}$

pos? ✓

9) $10^{2x-3} + 4 = 21$

$10^{2x-3} = 17$

$2x-3 = \log(17)$

$2x = \log(17) + 3$

$x = \frac{1}{2}(\log 17 + 3)$

or

$\frac{\log 17 + 3}{2}$

or

$\frac{\log 17}{2} + \frac{3}{2}$

10) $\log_6(x + 5) + \log_6 x = 2$

$\log_6(x^2+5x) = 2$

$x^2+5x = 6^2$

$x^2+5x-36=0$

$(x+9)(x-4)=0$

~~$x=-9$~~ $\boxed{x=4}$

11) $3 + 5^x = 21$

$5^x = 18$

$\boxed{x = \log_5 18}$

HW: Mixed Solving WS