

Warm-up: Identify the parent function. Identify all transformations and write the transformed parent table. Finally, rewrite the expression.

$$g(x) = 8(4)^{3-x} - 6$$

## Writing Exponential Equations

**MGSE9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).

**MGSE9-12.A.CED.2** Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**What am I learning today?**

How to write the equation for an exponential function

**How will I show that I learned it?**

Recognize an exponential scenario and write an equation to model it

An exponential function is created by  
**MULTIPLYING BY THE SAME NUMBER.**

It takes the general form of

$$f(x) = a(b)^{nx} + k$$

where **a** is the **initial value**, **b** is the **multiplier**,  
**x** is the **time**, **n** is the **frequency of change** and **k**  
is a **constant**.

Steps for writing exponential equations:

Step 1: Find the value you are starting with. Label this "a".

Step 2: Find your ratio of increase. If it is a %, change it to a decimal and add it to 1 if the problem is increasing or subtract it from 1 for decreasing. Label this "b".

Step 3: Plug into the formula  $y = A(B)^{nx}$

Step 4: When solving, pay attention to the rate of change. Divide your time by your rate to find the proper value of x.

Ex. A There are 10 mushrooms in your yard. The mushrooms double every 2 days.

a) Write an equation for this if  $x = \#$  of days.

$$a = 10$$

$$b = 2 \text{ (double)}$$

$$n = \frac{1}{2}$$

$$f(x) = 10(2)^{\frac{x}{2}}$$

b) How many mushrooms will there be in 15 days?

$$f(15) = 10(2)^{\frac{15}{2}}$$

$$176.4 \approx 1810.19$$

$$\approx 1810 \text{ mushrooms}$$

Ex. B A hand sanitizer kills  $\frac{1}{2}$  of the germs after each use. You have 5,000,000 germs on your hands now.

a) Write an equation for this if  $x = \#$  of uses.

$$a = 5,000,000 \quad f(x) = 5000000 \left(\frac{1}{2}\right)^x$$

$$b = 1 - \frac{1}{2} = \frac{1}{2}$$

b) How many germs will you have after 5 uses?

$$\begin{aligned} f(5) &= 5000000 \left(\frac{1}{2}\right)^5 \\ &= 156,250 \text{ germs} \end{aligned}$$

If there is a % change, the base is  $(1 + r)$  for growth and  $(1 - r)$  for decay.

What's going to be the base of your exponential in the following cases?

1. 20% increase

$$1 + 0.20 = 1.2^x$$

2. 4% decrease

$$1 - .04 = .96^x$$

3. 12.5% increase

$$1 + .125 = 1.125^x$$

4. 3.5% decrease

$$1 - .035 = .965^x$$

Ex. A The population of Atlanta was 420,000 in 2010 and expected to grow at a rate of 8% per decade. Predict the population, to the nearest ten thousand, for the year 2035.

$$a = 420,000$$

$$b = 1.08$$

$$n = \frac{1}{10}$$

$$P(t) = 420000(1.08)^{\frac{t}{10}}$$

$$t = 2035 - 2010 = 25$$

$$P(25) = 420000(1.08)^{\frac{25}{10}}$$

$$= 509106.54$$

$$\approx 510,000 \text{ people}$$

Ex. B Caffeine is eliminated from the bloodstream at a rate of 15% per half hour. There are initially 500 mg of caffeine. Predict the amount of caffeine in the bloodstream after 1 hour and 4 hours.

$$a = 500$$

$$b = 1 - .15 = .85$$

$$n = 2 \text{ (hours)}$$

$$C(t) = 500(.85)^{\frac{2t}{2}}$$

$$C(1) = 500(.85)^{\frac{2(1)}{2}}$$

$$= 361.25 \text{ mg}$$

$$C(4) = 500(.85)^{\frac{2(4)}{2}}$$

$$= 136.25 \text{ mg}$$

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$$n = \frac{1}{30} \text{ (minutes)}$$

$$C(t) = 500(.85)^{\frac{t}{30}}$$

$$C(60) =$$

$$C(240) =$$

## Steps for Writing Exponential Equations from a Table:

Step 1: Find the y-intercept. Label this "a".

Step 2: Find your ratio of change by dividing the 2nd # by the 1st #. Check it by dividing the 3rd # by the 2nd #. Label this "b".

Step 3: Plug into the formula  $y = A(B)^x$

**Your credit card debt is growing according to the following table.**

Month	Debt
0	-\$50.00
1	-\$52.00
2	-\$54.08
3	-\$56.24
4	-\$58.49

Write an equation to represent how much you owe based on how many months you haven't paid off the debt.

$$a = -50$$

$$b = \frac{-52}{-50} = 1.04$$

$$f(x) = -50(1.04)^x$$

$$(1.04)^{12} = 1.60 \rightarrow 60\% \text{ annual interest}$$

4% per month

**Your credit card debt is growing according to the following table.**

Month	Debt
0	-\$50.00
1	-\$52.00
2	-\$54.08
3	-\$56.24
4	-\$58.49

If you wait 1 year to pay off the debt, how much will you owe?  
How much additional money are you paying in interest?

$$f(12) = -50(1.04)^{12}$$

$$= -\$80.05$$

$$\text{Interest} = 80.05 - 50$$

$$= \$30.05$$

**Your credit card debt is growing according to the following table.**

Month	Debt
0	-\$50.00
1	-\$52.00
2	-\$54.08
3	-\$56.24
4	-\$58.49

B/t month  
10-12

rate = slope

When is your debt growing more quickly? Between months 1 and 3 or months 10 and 12?

$$\text{RoC}_{1-3} = \frac{-56.24 - (-52)}{3 - 1}$$

$$= -\$2.12 / \text{month}$$

$$(10, -74.01) \quad (12, -80.05)$$

$$\text{RoC}_{10-12} = \frac{-80.05 - (-74.01)}{12 - 10}$$

$$= -\$3.02 / \text{month}$$

Compound Interest Formula

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

$$P \left( 1 + \frac{r}{n} \right)^{t \cdot n}$$

A = final amount of money

P = Principal Investment (initial amount of money)

r = % change as a decimal

n = number of times compounded per year

t = number of years

annually  $n=1$   
 semi-annually  $n=2$   
 quarterly  $n=4$   
 monthly  $n=12$   
 daily  $n=365$

If you have \$5000 compounded quarterly at a rate of 6%, how much will you have after 5 years?  $n=4$   $r=.06$

$$A = 5000 \left( 1 + \frac{.06}{4} \right)^{4 \cdot 5} = \$6734.28$$

If you have \$5000 compounded monthly at a rate of 6%, how much will you have after 5 years?

$$A = 5000 \left( 1 + \frac{.06}{12} \right)^{12 \cdot 5} = \$6744.25$$

If you have \$5000 compounded monthly at a rate of 8%, how much will you have after 5 years?

$$A = 5000 \left( 1 + \frac{.08}{12} \right)^{12 \cdot 5} = \$7449.23$$

If you have \$5000 compounded monthly at a rate of 8%, how much will you have after 10 years?

$$A = 5000 \left( 1 + \frac{.08}{12} \right)^{12 \cdot 10} = \$11,098.20$$



## The number e

- Considered the "natural base" or the "Euler Number"

- Created by  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$

-  $\approx 2.72$  (expanded: 2.718281828459...)

## Continuously Compounded

$$A = \underline{Pe^{rt}}$$

A = final amount of money

P = Principal Investment (initial amount of money)

r = % change as a decimal

t = number of years

If you have \$5000 compounded continuously at a rate of 6%, how much will you have after 5 years?

$$A = 5000 \cdot e^{.06 \cdot 5} = \$6749.29$$

If you have \$5000 compounded continuously at a rate of 6%, how much will you have after 8 years?

$$A = 5000 \cdot e^{.06 \cdot 8} = \$8080.37$$